Differences between Quadratic Equations and Functions: Indonesian Pre-Service Secondary Mathematics Teachers' Views

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Differences between Quadratic Equations and Functions: Indonesian Pre-Service Secondary Mathematics Teachers' Views

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Abstract. Difference between quadratic equation and quadratic function as perceived by Indonesian pre-service secondary mathematics teachers (N = 55) who enrolled at one private university in Jakarta City was investigated. Analysis of participants' written responses and interviews were conducted consecutively. Participants' written responses highlighted differences between quadratic equation and function by referring to their general terms, main characteristics, processes, and geometrical aspects. However, they showed several obstacles in describing the differences such as inappropriate constraints and improper interpretations. Implications of the study are discussed.

1. Introduction

A plethora of studies indicates that there is relevance of teacher mathematical knowledge to high quality instruction and students' mathematics achievement [1, 2]. Teachers' knowledge of mathematics has become a remarkable headline and ongoing trend in mathematics education research for several decades now. Frameworks have been developed to understand teachers' knowledge of mathematics. It was the work of Ball and colleagues who firstly acknowledged the term of mathematics knowledge for teaching which has become one of the fundamental construction effort to comprehend knowledge required for teaching mathematics [3]. According to them, mathematics knowledge for teaching consisted of three types of subject matter knowledge (common content knowledge, specialized content knowledge, and horizon content knowledge) and three types of pedagogical content knowledge (knowledge of content and students, knowledge of content and teaching, and knowledge of curriculum). Even though they are categorized into distinct knowledge, sometimes certain cases could not be elucidated independently.

Teaching involves various activities such as showing students the way to address mathematical problems, answering students' posed questions, and monitoring their works [3]. It is common in mathematics classrooms that in dealing with their puzzlements, students pose questions to their teachers. The questions posed occasionally could not be predicted initially by teachers. For that reason, ability to anticipate what students are likely to think and ask and provide best responses to students' why questions are necessary for teachers. Subsequently, teachers are suggested to provide responses by leveraging decompressed mathematical knowledge that facilitates students to comprehend, refine, and develop their

knowledge. Many demands of teaching activities necessitate knowledge at the combination of student and contents. Therefore, teachers' mathematical knowledge for teaching is crucial for teaching.

The rationale behind the study is that there is scarcity in literature with respect to pre-service secondary mathematics teachers' views relative to discrepancies between quadratic equations and quadratic functions. The ground of why the author considered the topics was an interesting event occurred in peer teaching session. The peer teaching was one of the activities designed for the mathematics teaching development course. There was a pre-service teacher who tried to teach lesson segment of the concept of quadratic functions. At the beginning of the lesson, he was surprised by his friends who wondered about the differences between quadratic function and quadratic equations. Unfortunately, he could not make any significant and clear explications. The author then took this question into account.

As a matter of fact, the topics of quadratic equation and quadratic functions are presented in distinct area in secondary school mathematics curriculum. Quadratic equation precede quadratic functions as algebra is followed by function. Both are part of polynomials in which a large percentage of the middle and secondary mathematics curriculum is focused on by carrying out the operation and determining solutions or roots. The topics are cornerstone of subsequent mathematics topics such as trigonometry, limit, integral, and so forth. It is prevalent that in learning quadratic equation or quadratic function majority of students just memorize its rules devoid of reasoning and thinking deeply about them [4]–[6]. Previous studies highlighted students' lack of knowledge and misconceptions about factorization and the use of root statements [5]. In addition, students encounter adversity in articulating notion of variables and determining solution to a quadratic equation [7].

Students' difficulties and misconceptions in catching on both concepts are likely to be brought about by teachers' inability to transform and present mathematical knowledge properly. Brousseau, Sarrazy, and Novotná [8] acknowledged didactical obstacles, a term of which teachers transform a body of knowledge that might be different from the origin due to practical consideration. It appears to be the case that students' puzzlements in learning quadratic equations and quadratic functions might be attributed to several factors, one of which is pedagogical content knowledge of secondary mathematics teachers. Hence, teacher education is responsible for refining and developing pre-service teachers' mathematical knowledge in order to ensure high quality teaching.

The present study is substantial for several accounts. Recognizing pre-service secondary mathematics teachers' views about the differences could provide good opportunity for instructors to improve their performance in effort to improve and develop pre-service teachers' mathematical knowledge for teaching. In addition, findings of the present study could contribute to enriching body of knowledge of how pre-service secondary mathematics teachers' views concerning the differences between quadratic equations and quadratic functions. Besides, in-service mathematics teachers could benefit from pre-service teachers given responses regarding differences between quadratic equations and quadratic functions.

On the ground of the aforementioned discussion, the purpose of this study is to explore Indonesian pre-service secondary mathematics teachers' views about the differences between quadratic equations and quadratic functions. The study seeks explanation concerning their opinions as well as obstacles that pre-service teachers made when revealing their opinions.

Methods

Purposive sampling method was drawn upon to select participants. Participants of this study included fifty five pre-service secondary mathematics teachers who were enrolled as students at one private university in Jakarta City. They were in the third year at fourth-year secondary mathematics teacher education program. Data for the present study were collected in the spring of 2016-2017 academic year. At the time of the study, they have completed several teaching-related courses such as mathematics teaching and learning course and development of mathematics teaching course. Eight of the participants were male students, meanwhile the rest were female students.

2.1. Data Collection

Two sources of data were leveraged for this investigation: (1) participants' written responses; and (2) interviews. Participants' written responses were students' responses towards presented questions about the differences. The analysis of written responses is used to grasp participants' arguments. Subsequently, further investigation of participants' written responses was conducted by means of interviews. In this case, the author interviewed participants to gain deep information and justification directly concerning what they think and to conduct triangulation. Out of all participants, five of whom were interviewed whom the author thought would provide significant information. At the outset, protocol interview questions were prepared to guide the author during the interview.

The open-ended question that was asked to the participants, that is: *In case of a mathematics teacher introduces the concept of quadratic function (one-variable case), he/she reminds students about quadratic equations. Before continuing to subsequent explanation, one student interrupts and asks, "Sir/Mrs. what are the differences between quadratic equations and quadratic functions?" Suppose that you are the teacher, what will you explain to your students so that they understand its differences.* The question was given to the participants, and they were provided 100 minutes to write their argument concerning the differences. In addition, the question essentially required participants to describe the differences between quadratic equation and quadratic functions by means of their own so that students in secondary school level could grasp it easily.

2.2. Data Analysis

As an open-ended question was drawn on in the search of pre-service secondary mathematics teachers' views regarding the differences between quadratic equations and quadratic functions, qualitative approach was selected for the purpose of analyzing data. Categories and subcategories were constructed based on participants written responses. With respect to the reliability of categorization procedures, a second coder was involved. Agreement between coders for categorizing participants' responses was 94%. Triangulation method was conducted by interviewing five participants whose responses are interesting and in need of further and deep exploration.

3. Results and Discussion

In the following section, the findings will be presented in two major categories based on participants' responses concerning the question. The first category portrays participants' views about differences of quadratic equations and quadratic functions. This category consists of four sub-categories. The second category explicates participants' obstacles in describing the differences. In this category, there are four sub-categories constructed.

3.1. Participants' views about differences

Students' responses were likely to be influenced by th conceptions about algebra as Usiskin [9] described, namely, algebra as generalized arithmetic, algebra as the study of relationships among quantities, and algebra as the study of structures. In addition to it, we tried to describe participants responses in effort to shed light on the differences based on their written response by adapting conceptual framework for students understanding of logarithms proposed by Berezovski and Zazkis [10] and [11], namely, logarithm as object, logarithm as process, logarithm as function, and logarithm in contextual problems. The subcategories constructed are the differences based on the general terms, process, geometrical aspects, and main characteristics.

3.1.1. The differences based on its general forms

Out of 54 participants, 25 participants referred the differences to the general form of quadratic equation and quadratic functions. It means that the majority of responses fall into this subcategory. This response could be associated with the conception of algebra as generalized arithmetic. In this case, variables are wielded as generalizers in lieu of standing for specific unknown numbers [4]. According to Tanton [12], the general form of quadratic equations is and quadratic functions have general form in which a, b, and

c are either real or complex number and a is not zero. In addition to the general forms, several participants exemplified it by using numbers directly. Referring it to the general forms was highlighted on account of practical consideration in which students could easily discern the differences.

It is important to note that focusing solely on the general forms might lead students develop procedural or superficial knowledge devoid of conceptual or relational knowledge. Procedural knowledge, according to Hiebert and Lefevre [13], was described as knowledge concerning mathematical formal language or symbol representation system. In Indonesia, current mathematics curriculum is no longer centered solely on procedural skills; rather, it considers integration between procedural and conceptual development. Nevertheless, in fact procedural skills are still likely to be focus in most mathematics classrooms. Therefore, referring the differences based on the general forms is not enough and in need of being integrated with concept-based explanation.

3.1.2. The differences based on its main characteristics

Characteristics of equation and function were uncovered by twenty one participants in effort to differ the quadratic equations and functions. Tanton [12] described quadratic as any expression, function, or equation comprising variables raised to the second power, but no higher power. Several participants relied the difference on definition of quadratic similar to what Tanton [12] described. However, they made some additions such as explaining definition of equation and function. The equation was perceived by several participants as similarity between expression found in right side and left side. It is worth noting that participants' responses regarding equal sign converge with prior research [14] showing that students might develop various conceptions concerning equal sign, one of which is relational meaning. Relational meaning refers to various subcategories, such as formal equivalence describing equivalent terms [15]. Besides, several of them highlighted the definition of equation as an open sentence. The rationale behind the response is that a quadratic equation is a mathematical statement that can be either true or false relying on substituted values.

With respect to quadratic equation, several participants referred it to components of functions and related terms such as domain, codomain, range, dependent variables, and independent variables. They also emphasized function as a relation in which each component of the domain was mapped to exactly one component in the codomain. A participant expressed that quadratic function consisted of two distinct variables, that is, x and $f(x)$ or y.

3.1.3. The differences based on process

There were five teen participants who viewed that quadratic equations and quadratic functions as processes. According to them, quadratic equations referred to process to determine its roots. It means that the participants say that quadratic equation automatically led them to determine the value of x. It fits with conception of algebra as study of procedures for solving certain kinds of problems [9]. They were likely to consider that quadratic equation and quadratic functions as problem that should be addressed. Just as logarithm [16], quadratic equation and quadratic functions had its operational characteristics and complex process.

Factorization and quadratic formula were two methods that were mostly mentioned by participants attempt to determine the value of x or solving quadratics equation. Only two participants mentioned completing the square as one method to determine the roots of quadratic equation. Several participants completed their explanation presenting direct examples of how to solve quadratic equations. To solve it effectively and correctly, mastering topics such as algebra, fractions, negative numbers and algebraic expansions are required [17]. In short, algebraic manipulations are perceived as the most important aspect in dealing with quadratic equations.

Meanwhile according to the participants, quadratic functions referred to process of fixing certain value of x or domain of function and determining the value of function or range of functions. This conception is likely to be in line with what Usiskin [9] shed light on the conception of variables as argument or parameter that have distinct values and relate to other variables. In this case, the value of y

as dependent variable is determined by the value of x as independent variable. Therefore both values are related each other.

3.1.4. The differences based on geometric aspects

Thompson and Chappell [18] revealed that representation is a substantial component in the elaboration of mathematical literacy. Therefore, in addition to dealing with symbolic manipulation, geometric aspects were considered by twelve participants as quadratic functions referred to constructing graphic or curves. The form of curve that is constructed is parabola. One participant wrote that in attempt to draw graphic, the initial step was substituting dependent variable (x) by specific number and subsequently determining points on Cartesian coordinates. Another response was determination of intersection point with x or y axis. Concerning quadratic equation, one participant claimed that there was no graphic or curve that could be constructed to stand for quadratic equation. Nevertheless, another participant drew quadratic equation as one or two straight lines that intersected x axis. Referring to the general form of quadratic equation which consists of one unknown or variable solely and disregarding another variable, argument expressed that there was no representation of quadratic equation could be retained. Considering visual representation of quadratic functions might lead students to learn mathematics in a meaningful manner.

3.2. Participants' Obstacles

In attempt to uncover the discrepancies between quadratic equations and quadratic functions, several participants indicate obstacles in describing it, that is, inappropriate constraints and improper interpretation of the general forms.

3.2.1. Inappropriate constraints

Even though the majority of the participants highlighted the differences based on its general forms, several of them were unable to show appropriate constraints. The following are participants' deficiencies in articulating general forms of quadratic equations and quadratic functions properly.

- Confusions between real and constant number in describing the value of a, b, and c in general form of quadratic equation and functions.
  There were several participants expressed that in quadratic equation the value of a, b, and c were real numbers, yet in the following explanation he/she considered that in quadratic function, the value of a, b, and c were constants or vice versa. Some of them argued that the value of a, b, and c were constant real numbers. It is not surprising since literature also indicate discrepancies in illuminating the value of a, b, and c in which most of literature refer it to real numbers [12], meanwhile the others refer it to constants [19]. Even though they are expressed in a distinct manner, as a matter of fact, a a, b, and c are known or given. In addition, a constant is a specific real number. Nevertheless, consistency is required in describing both quadratic equation and quadratic functions. Besides, confusion relative to the value of x was demonstrated by several participants. They argued that the value of x was not equal to zero. It seems that they have puzzlement in determining which variables whose value is zero.
- Incomplete constraints
  Several participants did not get across the constraints of the general terms completely. The most two common mistakes are that the value of a is not determined and the value of a, b, and c are not specified succinctly. Determining the value of a is important in describing quadratic equation and function.

  This obstacle might be attributed to mathematics teaching process in which teachers might focus solely on its general form of quadratic equation and function, yet they might not write completely with the constraints or explain it further. Getting rid of constraints and ignoring deep and further explanation of its general terms seems to be frequently found in mathematics classrooms.

3.2.2. Improper Interpretations

Improper interpretation is a common and persistent problem in algebra learning. In this case, for example, several participants interpret the general form of quadratic equation and quadratic functions improperly such as:

a. Quadratic equation is algebraic statement whose result is zero, meanwhile the result of quadratic function is not zero.
b. Quadratic equation is a mathematical statement which has equal sign and has value of zero, meanwhile the value of quadratic function can be zero and non-zero.
c. Quadratic equation has equal sign, whereas quadratic function does not have.

These obstacles might stem from participants' inability to give other explanations concerning the differences. In addition, it might occur as a result of teaching strategies that present mathematics concept or topics separately. Therefore, they made comments based on the general forms by their own. Mathematics concepts or topics should be taught in an integrated manner. At the beginning of lesson, teachers are suggested to review previous concepts and make connection to new concepts. Ignoring related previous concepts might lead students to interpret the concept improperly. According to the above participant's responses, the statement a and statement b are very succinct that participants make improper interpretation. Direct interpretation devoid of considering other mathematical concepts lead participants to make over-generalizations. On the other hand, the 3rd statement stems from participants' understanding of another written form of function, that is, $f: x \rightarrow f(x)$ and she concluded that the general form of quadratic functions did not have equal sign.

4. Conclusion

A study conducted by Vaiyavutjamai and Clements [20] showed that common problem encountered in middle grade was that even though students were able to obtain correct solution in determining roots of quadratic equations, they had serious puzzlements concerning the actual meaning of quadratic equations. It might be due to teaching strategies which emphasize heavily on procedural knowledge in lieu of integration of procedural and conceptual knowledge or on certain type of representation instead of multiple representations. Therefore, when mathematics teachers are asked by students who have difficulty in grasping certain concept, the teachers should provide appropriate responses by considering various aspects or representations. In addition, teachers are in need of decompressing mathematical knowledge in such that students grasp it easily and properly.

The written responses of the participants relative to differences between quadratic equation and quadratic function provided sufficient insights for us to consider various ways to deal with it. The results thus far show that participants referred the differences to their general forms, main characteristics, processes, and geometric aspects. Use of multiple aspects and representations in unpacking the differences might help both students and teachers in dealing with the topics. Besides, in expressing their responses several participants demonstrated obstacles such as inappropriate constraints and improper interpretation. The reason might lay in the fact that instruction or learning strategies emphasize heavily on symbolic manipulation in lieu of concepts and meaning.

The present study will assist mathematics teacher educators in obtaining information on pre-service secondary mathematics teachers' view and obstacles concerning quadratic equations and quadratic function. For the purpose of coming to grips with the obstacles, teacher educators should be more aware of pre-service teachers' mathematical knowledge and learning. The findings could be a reference for teacher educators to seek alternative methods in dealing with pre-service teachers' obstacles especially in the topics. Further studies might focus on investigation of pre-service secondary mathematics teachers' views on similarity and relations between quadratic equations and quadratic function.

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Differences between Quadratic Equations and Functions: Indonesian Pre-Service Secondary Mathematics Teachers' Views

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