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Investigating fraction computation problem-solving among pre-service primary school teachers

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Abstract

The ability to solve problems involving fractions is a fundamental aspect of mathematics education. This study explores how Pre-Service Primary School Teachers approach problem-solving in fractional computations. A workbook is designed to support pre-service primary school teachers ' computational thinking in fraction-context challenges. The study employs a qualitative descriptive method encompassing interviews, study documentation, and observation to assess fraction computation problem-solving abilities. Twenty-seven participants were involved in this study. They were first-year students enrolled in the Primary School Teacher Education Department at one of the private universities in Jakarta, Indonesia. The findings reveal a notable outcome in pre-service primary school teachers ' understanding of fraction computation problem-solving, marked by recognizable strategies in their problem-solving approach. This research suggests that designing a series of workbooks containing various strategies in computational fractions and building a strong fractional number sense can help pre-service teachers reduce misconceptions and better understand fraction operations. These findings offer guidance for mathematics teacher education on how to effectively teach and embed the concept of fraction calculations to their future students so that they can only teach procedurally if they understand the meaning of fraction operations.

Keywords: calculation; fraction; pre-service primary school teacher; workbook

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Introduction

Understanding fraction computation is essential in mathematics teaching, especially for pre-service primary school teachers. Fractions are complex and serve as a basis for comprehending other number kinds and algebraic operations in later school years (Duodu et al., 2019). Future educators play an essential role in establishing young learners' mathematical foundations, and their competency with fraction calculation directly impacts their capacity to teach this idea effectively.

Despite the centrality of fractions in the primary school curriculum, research shows that many pre-service teachers need help with fraction problem-solving, which can lead to misconceptions and ineffective instruction in their future classrooms. Bowie et al. (2019) and Sin (2021) revealed that pre-service teachers possess a limited understanding of various fraction interpretations and need more proficiency in explaining the procedures for adding and subtracting fractions. Their familiarity with the particular meanings of fractions could be improved. They are more acquainted with the part-whole sub-construct compared to other sub-constructs. Moreover, the ability to identify and address common errors and misconceptions in fraction computation is a key component of effective mathematics teaching. Pre-service teachers must be equipped with strategies to recognize students' typical mistakes and understand the underlying misconceptions that lead to these errors. Research indicates that pre-service teachers need help understanding fractions and the concept of dividing fractions (Ball, 2021). They need to gain comprehension of the operator construct of rational numbers. Silver and Lesh (2016) face challenges in explaining fractions and the reasoning behind algorithms to children (Chinnappan, 2000). Additionally, while they may arrive at correct answers, they often fail to execute fractional computation procedures accurately (Huang et al., 2013).

Computing and solving fraction problems takes more than just procedural knowledge. It also needs a thorough understanding of the underlying mathematical principles and the ability to apply that knowledge in various circumstances. Computation in fractions includes skills like addition, subtraction, multiplication, and division of fractions, as well as the ability to simplify fractions and convert improper fractions to mixed numbers. These abilities are necessary for thoroughly understanding rational numbers and their applicability in real-world circumstances. Olanoff et al. (2014) reviewed some articles examining prospective teachers' fraction knowledge. They discovered that while prospective teachers are relatively proficient in performing procedural tasks, they generally lack the flexibility to deviate from these procedures and apply "fraction number sense."

However, Kolar et al. (2018) discovered that prospective teachers struggled more with procedural comprehension than a conceptual understanding of fractions when comparing them. While students understood the significance of a fixed whole in real-world circumstances, they needed help with the proper processes for comparing fractions when comparing two numbers. According to Dita and Abate (2023), the problem-solving abilities of pre-service primary school teachers in the context of fraction computation are critical for various reasons. For starters, it sheds light on the current status of mathematics competence among potential teachers, highlighting areas of strength and indicating deficiencies that must be addressed in teacher education programs. Second, knowing the unique issues faced by pre-service teachers can inform the creation of specialized interventions and instructional practices.

Finally, improving pre-service teachers' fraction computation skills can increase the overall quality of mathematics education in elementary schools.

This study aims to look into how pre-service primary school teachers solve fraction computation problems. It specifically aims to assess their ability to perform fraction operations, uncover common errors and misconceptions, and investigate individuals' strategies to solve fraction problems. We expect that our inquiry will add to the corpus of information on mathematics teacher education and provide recommendations for strengthening the mathematical preparation of future primary school teachers since many teachers in Indonesia still use procedural methods in fraction operations without understanding the underlying meaning of fractions themselves (Pramudiani et al., 2024). Furthermore, in the study of Pramudiani et al. (2024), they prefer using procedural methods because they follow what was taught by their teachers in primary school. Based on the theories and models used in previous research, the researchers will discuss the importance of conceptual and procedural knowledge in mathematics education (Hakim & Yasmadi, 2021). This issue requires attention, as it is essential for teachers to comprehend the meaning behind numerical operations, including fractions, to convey meaningful concepts to their students effectively.

Therefore, this study aims to investigate the fraction computation problem-solving abilities of pre-service primary school teachers. Specifically, it examines their proficiency in performing fraction operations, identifies common errors and misconceptions, and explores the strategies they use to solve fraction problems. Through this investigation, we hope to contribute to the body of knowledge on mathematics teacher education and provide recommendations for improving the mathematical preparation of future primary school teachers.

Methods

This qualitative study was conducted to analyze the fraction computation of pre-service primary school teachers. A qualitative methodology known as narrative research is derived from written or spoken texts that recount stories of related occurrences chronologically (Czarniawska, 2004). One way to define narrative research is as a methodology, examining personal experiences as a source of knowledge in and of itself that calls for further comprehension (Nasheeda et al., 2019). Twenty-seven participants were involved in this study. They were first-year students enrolled in the Primary School Teacher Education Department at one of the private universities in Jakarta, Indonesia.

The used instrument contains algebraic computation of fractions with five types of questions designed in a series of workbooks (Table 1). The research techniques for gathering data included interviews and focus group discussions, study documentation, and observation toward assessing fraction computation problem-solving abilities.

Table 1. Designed workbook

<table>
<tr><th>Code</th><th>Questions (in Bahasa)</th><th>Questions (in English)</th></tr>
<tr><td>QA</td><td>
Selesaikan tiga soal berikut ini.

$1\frac{7}{15} + 45 =$

$5\frac{2}{3} + \frac{7}{11} =$

$\frac{7}{8} + 4\frac{5}{24} =$

Andaikan kamu adalah seorang guru kelas 5 SD. Kamu ingin menjelaskan kepada siswa/i bagaimana kamu menyelesaikan soal berikut. Jelaskan strategi/cara mu menyelesaiakan soal berikut yang dapat dimengerti oleh siswa kelas 5 SD.
</td><td>
Solve the following three problems.

$1\frac{7}{15} + 45 =$

$5\frac{2}{3} + \frac{7}{11} =$

$\frac{7}{8} + 4\frac{5}{24} =$

Imagine you are a teacher of a 5th grade class. You want to explain to a student how you solve these problems. Describe your strategy in a way that a 5th grade student understands what you do.
</td></tr>
<tr><td>QB</td><td>
$5\frac{2}{7} + 3\frac{1}{5} = 8 + \frac{2}{7} + \frac{1}{5}$ benar atau salah

$7\frac{4}{9} - 3\frac{1}{3} = 4 + \frac{4}{9} - \frac{1}{3}$ benar atau salah

$2\frac{1}{5} + 3\frac{1}{4} = 5 + \frac{1}{5} + \frac{1}{4}$ benar atau salah
</td><td>
$5\frac{2}{7} + 3\frac{1}{5} = 8 + \frac{2}{7} + \frac{1}{5}$ true or false

$7\frac{4}{9} - 3\frac{1}{3} = 4 + \frac{4}{9} - \frac{1}{3}$ true or false

$2\frac{1}{5} + 3\frac{1}{4} = 5 + \frac{1}{5} + \frac{1}{4}$ true or false
</td></tr>
<tr><td>QC</td><td>
Anjar, Haby, Citra, dan Wahu belajar kelompok. Pertama, mereka menyelesaikan secara individu $2\frac{1}{5} + 3\frac{1}{4} = 5 + \frac{1}{5} + \frac{1}{4}$ benar atau salah. Masing-masing dari mereka sudah menyelesaikan soal tersebut dengan caranya masing-masing, lalu mereka berdiskusi hasil jawaban mereka.

Anjar:

$2\frac{1}{5} + 3\frac{1}{4} = 5 + \frac{1}{5} + \frac{1}{4}$ adalah benar.

Pertama, hitung $2\frac{1}{5} + 3\frac{1}{4}$.

$2\frac{1}{5} + 3\frac{1}{4} = \frac{11}{5} + \frac{13}{4} = \frac{44}{20} + \frac{65}{20} = \frac{109}{20} = 5\frac{9}{20}$

Kemudian, hitung $5 + \frac{1}{5} + \frac{1}{4} = 5 + \frac{4}{20} + \frac{5}{20} = 5\frac{9}{20}$.

Hasil dari ruas kanan dan kiri sama, sehingga pernyataannya bernilai benar.

Haby:

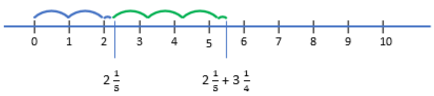
$2\frac{1}{5} + 3\frac{1}{4} = 5 + \frac{1}{5} + \frac{1}{4}$ benar. Haby menulis kembali pernyataannya.

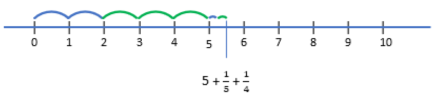
$2\frac{1}{5} + 3\frac{1}{4} = 2 + \frac{1}{5} + 3 + \frac{1}{4} = 2 + 3 + \frac{1}{5} + \frac{1}{4} = 5 + \frac{1}{5} + \frac{1}{4}$.

Sehingga, pernyataan ini bernilai benar.

Citra:

$5\frac{2}{7} + 3\frac{1}{5} = 8 + \frac{2}{7} + \frac{1}{5}$ adalah benar. Saya menggunakan garis bilangan. Saya menggambar bagian pertama $2\frac{1}{5} + 3\frac{1}{4}$.





Bagaimana menurutmu dengan yang dikerjakan oleh Anjar, Haby dan Citra? Bagaiamana pendekatan berbeda yang mereka kerjakan? Apakah pendekatan mereka sama?
</td><td>
Anjar, Haby, Citra, and Wahu work together. First, they individually solved the statement $2\frac{1}{5} + 3\frac{1}{4} = 5 + \frac{1}{5} + \frac{1}{4}$ true or false. When all four students had answered the question, they compared their work.

Anjar:

$2\frac{1}{5} + 3\frac{1}{4} = 5 + \frac{1}{5} + \frac{1}{4}$ is true.

First, I calculated $2\frac{1}{5} + 3\frac{1}{4}$.

$2\frac{1}{5} + 3\frac{1}{4} = \frac{11}{5} + \frac{13}{4} = \frac{44}{20} + \frac{65}{20} = \frac{109}{20} = 5\frac{9}{20}$.

Next, I calculated $5 + \frac{1}{5} + \frac{1}{4} = 5 + \frac{4}{20} + \frac{5}{20} = 5\frac{9}{20}$.

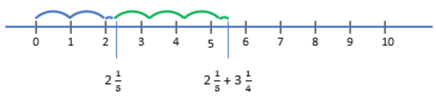
The two sides are the same, so the statement is true.

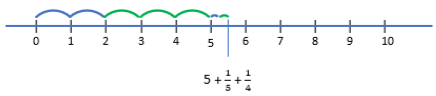
Haby:

$2\frac{1}{5} + 3\frac{1}{4} = 5 + \frac{1}{5} + \frac{1}{4}$ is true. I rewrote the statement $2\frac{1}{5} + 3\frac{1}{4} = 2 + \frac{1}{5} + 3 + \frac{1}{4} = 2 + 3 + \frac{1}{5} + \frac{1}{4} = 5 + \frac{1}{5} + \frac{1}{4}$. So, the statement is true.

Citra:

$5\frac{2}{7} + 3\frac{1}{5} = 8 + \frac{2}{7} + \frac{1}{5}$ is true. I use a numberline. I draw the first part $2\frac{1}{5} + 3\frac{1}{4}$.





What do you think of Anjar's, Haby's and Citra's ideas? How are their approaches different, how are their approaches the same?
</td></tr>
</table>

QD (a)	Selesaikan soal berikut menggunakan pendekatan Anjar, Haby, Citra dan Wahu. $3\frac{1}{3}-2\frac{1}{4}=1+\frac{1}{3}-\frac{1}{4}$ benar atau salah		Solve these two problems using the three approaches from Anjar, Haby, Citra and Wahu. $3\frac{1}{3}-2\frac{1}{4}=1+\frac{1}{3}-\frac{1}{4}$ true or false	
	Pendekatan Anjar		Anjar's Approach	
	Pendekatan Haby		Haby's Approach	
	Pendekatan Citra		Citra's Approach	
	Pendekatan Wahu		Wahu's Approach	
QD (b)	$6\frac{1}{15}-3\frac{1}{5}=3\frac{4}{15}$ Benar atau salah		$6\frac{1}{15}-3\frac{1}{5}=3\frac{4}{15}$ true or false	
	Pendekatan Anjar		Anjar's Approach	
	Pendekatan Haby		Haby's Approach	
	Pendekatan Citra		Citra's Approach	
	Pendekatan Wahu		Wahu's Approach	
QE	Berikan ide dari Anjar, Haby dan Citra bagaiamana kamu sekarang menyelesaikan soal-soal berikut. $2\frac{11}{14}-1\frac{1}{7}=$ $7\frac{2}{11}-\frac{1}{2}=$ $4\frac{11}{12}-2\frac{5}{8}=$		Given the ideas of Anjar, Haby, and Citra how would you now solve the following problems. $2\frac{11}{14}-1\frac{1}{7}=$ $7\frac{2}{11}-\frac{1}{2}=$ $4\frac{11}{12}-2\frac{5}{8}=$	

In the designed workbook (Table 1), the pre-service primary school teachers were given several problems subsequently. First, they were asked to solve the addition of mixed fractions, and they had to imagine that they were a teacher of a 5th-grade class. In this stage, they were asked to solve three problems related to adding mixed fractions. They were then required to explain how they solved these problems and describe strategies or methods to solve them in a way that fifth-grade students could understand. Second, they were given another fraction computation problem with true and false questions. In this stage, they were asked to work in pairs and to determine whether adding mixed fractions was correct or incorrect, including their reasoning.

In the third stage, the pre-service primary school teachers were given an illustration of a study group consisting of Anjar, Haby, Citra, and Wahu. In this study group, they solved the problems individually first and then discussed their answers. Based on the illustration, they were asked to answer the questions such as: *"What do you think about the work done by Anjar, Haby, and Citra? What approach did they use? Are the approaches they used the same? And is there any student who can solve the fraction problem using contextual situations like Wahu?"*

In the fourth stage, the pre-service primary school teachers were required to solve true or false problems based on the illustration analysis (using the approaches of Anjar, Haby, Citra, and Wahu). Finally, in the fifth stage, they were asked to solve mixed fractions problems based on the illustration analysis (using the approaches of Anjar, Haby, Citra, and Wahu), and they were evaluated on how they would solve the three problems related to the subtraction of mixed fractions.

In addition to taking a written test, the pre-service teachers were also interviewed. Throughout the interview, they were asked several questions regarding their answer sheets, both individually and in group discussions. Follow-up interviews were employed by the researchers to assist in defining themes and concepts in the interviewees (Kwarteng and Ahia, 2015).

Results

The design of this task was tested on prospective teachers in the Elementary School Teacher Education Department through five questions presented in groups, as seen in Table 1. This task was designed to determine prospective teachers' ability to solve mixed fraction problems and apply steps in working on mixed fractions.

Question type A: Mixed fraction

Selesaikan tiga soal berikut ini. $1\frac{7}{15} + 45 =$ $5\frac{2}{3} + \frac{7}{11} =$ $\frac{7}{8} + 4\frac{5}{24} =$ Andaikan kamu adalah seorang guru kelas 5 SD. Kamu ingin menjelaskan kepada siswa/i bagaimana kamu menyelesaikan soal berikut. Jelaskan strategi/cara mu menyelesaiakan soal berikut yang dapat dimengerti oleh siswa kelas 5 SD.	Solve the following three problems. $1\frac{7}{15} + 45 =$ $5\frac{2}{3} + \frac{7}{11} =$ $\frac{7}{8} + 4\frac{5}{24} =$ Imagine you are a teacher of a 5th grade class. You want to explain to a student how you solve these problems. Describe your strategy in a way that a 5th grade student understands what you do.

Figure 1. Question type A number 1-3

This type of question consists of three questions that require students to work on adding mixed fractions with their methods. The questions can be seen in Figure 1. In general, the steps used by the students were as follows: The first thing to do was to change the mixed fraction into an improper fraction before performing the arithmetic operation. This process involves multiplying the integer by the denominator of the fraction. After the mixed fraction was changed into an improper fraction, students continued by finding the Least Common Multiple (LCM) of the fractions. After that, they multiplied the numerator with the same number needed to get the common denominator and performed the addition or subtraction operation. In the next stage, they turned the fraction into a mixed number, and some simplified the new fraction. Based on the student answer sheets obtained, the next step was to carry out an analysis based on the steps or approaches used by students in the fraction questions. Generally, in this question, students are expected to be able to solve the three questions more effectively and precisely. The results of the analysis of student answers based on the stages of students' approaches to fraction problems are as follows:

1. Convert the mixed numbers to improper fractions (C);
2. Find the Least Common Multiple (LCM) of the fractions (L);
3. Multiply the numerator with the same number needed to get the common denominator (M);
4. Operate numerators (O);
5. Turn the fraction into a mixed number (T);
6. Simplify the new fraction (S).

Based on the stages above, it can be seen that students' answers can be classified differently for various reasons which are described as follows.

Question type A number 1

The Question in number 1 type A shows the results of students' answers to mathematical fraction questions. Fraction question number 1 shows that 25 students answered from the first to the fifth stage (convert the mixed numbers to improper fractions, find the LCM of the fractions, multiply the numerator with the same number needed to get the common denominator, operate numerators, and turn the fraction into a mixed number). One example of the student's answer is as follows in Figure 2.

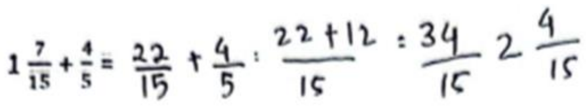


Figure 2. The answer of Q1 using CLMOT by S1

From the answer in Figure 2, it can be indicated that student understands the concept of addition in fractions with different denominators. The steps taken were to simplify the mixed fraction $1\frac{7}{15}$ to $\frac{22}{5}$. After that, he wrote the answer $\frac{22}{5}$ plus $\frac{4}{5}$. Then, he looked for the LCM to equate the denominators. The LCM sought was 15 and 5, so that the number 15 was obtained. Then, 15 was divided by 15, resulting in 1, which was then multiplied by 22 to get 22. Then, 15 was divided again by 5, resulting in 3, which was then multiplied by 4 to get 12. As a result, 22 was added to 12, giving 34, and the fraction was expressed as $\frac{34}{15}$. Then, the fraction was simplified into a mixed fraction which gets the result $2\frac{4}{15}$. From S1's answer, it can be seen that S1 has equated the denominators, found the LCM, added the numerators and obtained the correct result, which is actually a procedural method commonly taught in schools and typically outlined in textbooks.

Meanwhile, 1 student answered from the first to the sixth stage (convert the mixed numbers to improper fractions, find the LCM of the fractions, multiply the numerator with the same number needed to get the common denominator, operate numerators, turn the fraction into a mixed number, and simplify the new fraction). However, the answer is not entirely accurate. It can be seen in Figure 3 below:

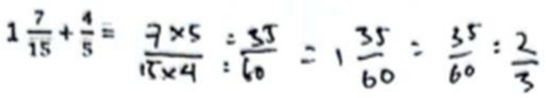


Figure 3. The Answer of Q1 Using CLMOTS by S24

Based on the solution steps above, it can be seen that S24 has implemented a strategy by making the denominators of both forms of fractions the same by applying cross multiplication $\frac{a}{b}+\frac{a}{b}$. The cross multiplication performed is the left numerator 7 multiplied by the right denominator 5, and the left denominator 15 multiplied by the right numerator 4. So, the result obtained from the multiplication is $\frac{35}{60}$. Then, the ordinary fraction was simplified to $\frac{2}{3}$ which was wrong. Based on the approach proposed by S24, he actually gave the wrong procedure in the part of equating the denominators. So, when it was wrong at the beginning, the final answer was definitely wrong. This means that students did not fully understand the steps in working on mixed fractions, but this student has made an effort to reach the sixth stage (simplifying the new fraction).

Question type A number 2

Fraction question number 2 type A shows that 25 students answered from the first stage to the sixth stage, including convert the mixed numbers to improper fractions (C), find the LCM of the fractions (L), multiply the numerator with the same number needed to get the common denominator (M), operate numerators (O), and turn the fraction into a mixed number (T), and simplify the new fraction (S). One example of the student's answer is as follows in Figure 4.

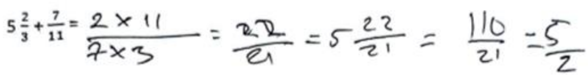


Figure 4. The answer of Q2 using CLMOTS by S24

Based on the solution steps above, it can be seen that the student has implemented a strategy by making the denominators of both forms of fractions the same by applying cross multiplication $\frac{a}{b}+\frac{a}{b}$ namely the right numerator 2 multiplied by the left denominator 11, then the left denominator 3 multiplied by the right numerator 7. So, it was written as $\frac{2\times11}{7\times3}$. From the cross multiplication obtained the result $\frac{22}{21}$. Then, S24 added the number 5 to become $5\frac{22}{21}$. From the mixed fraction he turned it into an ordinary fraction to $\frac{110}{21}$, then simplified it again to $\frac{5}{2}$. Similarly to the answer in question 1, the approach proposed by S24 was wrong in the part of equating the denominators. So, when it was wrong at the beginning, the final answer was definitely wrong. This means that students did not fully understand the steps in working on mixed fractions, but this student has made an effort to reach the sixth stage (simplifying the new fraction).

Next, the answers from the other students are as shown in Figure 5 below.

$$5\frac{2}{3}+\frac{7}{11}= \frac{17}{3}+\frac{7}{11} = \frac{187+21}{33} = \frac{208}{33} = 6\frac{10}{33}$$

Figure 5. The Answer of Q2 Using CLMOT by S3

From the answer above, the steps used were first S3 simplified mixed fraction into ordinary fraction from $5\frac{2}{3}$ to $\frac{17}{3}$. After that, S3 found the LCM of 3 and 11 which got the result 33 to make the denominators the same. Then, S3 added the numerators and got the result $\frac{208}{33}$. From the fraction, he then simplified the fraction into a simpler number to $6\frac{10}{33}$. From the answer it can be seen that students looked for the LCM, added the numerators and got the correct result. The number of students who answered using this strategy was 18 out of 27 students. This shows that students understand the concept of addition of fractions with different denominators procedurally.

Furthermore, 1 student worked on the fraction problem in a different way as seen in Figure 6 below.

$$5\frac{2}{3}+\frac{7}{11}= 5\,\frac{22+21}{33} = 5\,\frac{43}{33}\ (43-33)$$
$$= 6\frac{10}{33}$$

Figure 6. The Answer of Q2 Using CLMOT by S15

From the Figure 6 above, it can be seen that S15 used a different strategy. S15 used a method of subtraction between the numerator and denominator (43-33). After that, he added 1 to the integer 5, resulting in 6, and wrote the remaining fraction as $\frac{10}{33}$. From this strategy, he got the result to be $6\frac{10}{33}$.

Question type A number 3

Fraction question number 3 type A shows that 25 students answered from the first stage to the sixth stage, including convert the mixed numbers to improper fractions (C), find the LCM of the fractions (L), multiply the numerator with the same number needed to get the common denominator (M), operate numerators (O), and turn the fraction into a mixed number (T), and simplify the new fraction (S). One example of the student's answer is as follows in Figure 7.

$$\frac{7}{8}+4\frac{5}{24}= \frac{7}{8}+\frac{101}{24} = \frac{21+101}{24} = \frac{122}{24}\ \ 5\frac{2}{24} = \frac{1}{12}$$

Figure 7. The answer of Q3 using CLMOTS by S1

Based on the solution steps above in Figure 7, it can be seen that the student has implemented a strategy by finding the LCM to equate the denominators, namely by finding the

LCM between 8 and 24. Then, he added the numerators, so that it became $\frac{21+101}{24} = \frac{122}{24}$. Then he changed it to a mixed fraction back to $5\frac{2}{24}$, and simplified it to an ordinary fraction $\frac{1}{12}$. The problem worked on by S1 actually gave the wrong answer, because he did not bring back the number 5, but there was an attempt to reach the sixth stage, namely changing it to an ordinary fraction even though in this problem it could not be an ordinary fraction. Furthermore, for the answers from other students as in Figure 8 below.

$$\frac{7}{8}+4\frac{5}{24} = \frac{7}{8}+\frac{101}{24} = \frac{168+808}{192} = \frac{976}{192} = \frac{244}{48} = \frac{61}{12} = 5\frac{1}{12}$$

Figure 8. The answer of Q3 using CLMOTS by S3

Based on the Figure 8 above, it can be indicated that S3 used the idea of addition in fractions with different denominators. From the answers it can be seen that equating the denominators to 192, adding the numerators from 168 + 808 to 976, and getting the correct result, namely $\frac{976}{192} = \frac{244}{48} = \frac{61}{12}$, then S3 changed the improper fraction into a mixed fraction $5\frac{1}{12}$. Students who answered using this strategy numbered 17 out of 27 students. This shows that students solved the problem of addition of fraction procedurally. Furthermore, 1 student solved the fraction problem in a different way as can be seen in Figure 9 below.

$$\frac{7}{8}+4\frac{5}{24} = 4\frac{40+31}{24} = 4\frac{71}{24} \quad (71-24)$$
$$= 4\frac{18}{24}$$

Figure 9. The answer of Q3 using CLMOT by S15

Based on the answer of S15 in Figure 9, it can be seen that S15 used a different strategy. S15 used a method of subtraction between the numerator and denominator (71-24). However, it was different with what he did in Q2A, he did not add the subtraction result to the integer, so the integer remains 4, and he got the wrong result subtraction of fraction, namely $4\,\frac{18}{24}$. To sum up, from these data, it shows that almost all students were able to solve mathematical fraction problems. However, they used a procedural approach.

Question type B: True and false

This type of question involves determining whether statements are true or false. In this type of question, the students must choose one of the two options after analyzing and proving the given answer. If the answer to the question is correct and the student answers correctly, it means the student has understood the question, along with the strategies, well. Conversely, if the question is correct but the student answers incorrectly, then the student has not understood the question and the strategies fully. Likewise, if the answer to the question is incorrect and the student answers correctly, it means that the student has not understood the question and the strategies fully. However, if the answer to the question is wrong and the student answers incorrectly, then he has understood the question correctly. Figure 10 below is a Type B question (true-false):

$5\frac{2}{7} + 3\frac{1}{5} = 8 + \frac{2}{7} + \frac{1}{5}$ benar atau salah	$5\frac{2}{7} + 3\frac{1}{5} = 8 + \frac{2}{7} + \frac{1}{5}$ true or false
$7\frac{4}{9} - 3\frac{1}{3} = 4 + \frac{4}{9} - \frac{1}{3}$ benar atau salah	$7\frac{4}{9} - 3\frac{1}{3} = 4 + \frac{4}{9} - \frac{1}{3}$ true or false
$2\frac{1}{5} + 3\frac{1}{4} = 5 + \frac{1}{5} + \frac{1}{4}$ benar atau salah	$2\frac{1}{5} + 3\frac{1}{4} = 5 + \frac{1}{5} + \frac{1}{4}$ true or false

Figure 10. Question type B number 1-3

Based on the students' answer sheets, there are various answers provided by the students with different reasons outlined as follows in Figure 11:

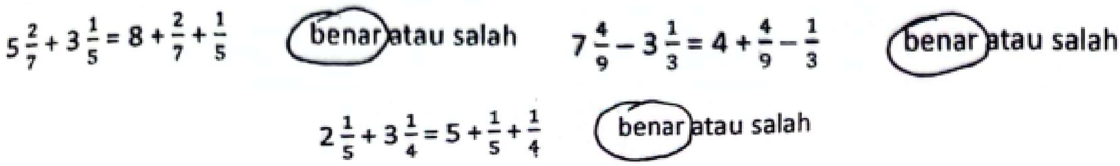


Figure 11. The answer of QB by S1

Based on the answer of S1 (Figure 11), the student chose the "true" option without providing any strategy of his work. Then, there was other student who answered using the strategies such as follows in Figure 12:

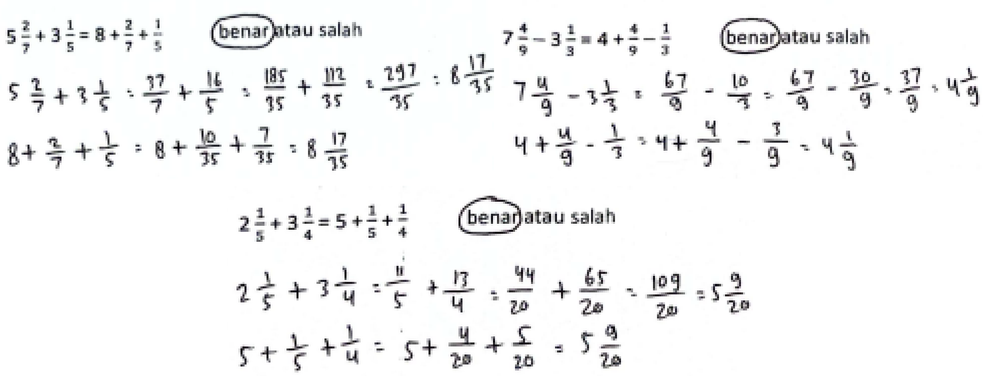


Figure 12. The answer of QB by S13

Based on the answer of S13 (Figure 12), she chose the correct option for the problem. To prove their work, she followed CLMOT strategy and then, she equated the two given fractional expressions. Next, for the answer of other student revealed as follows in Figure 13:

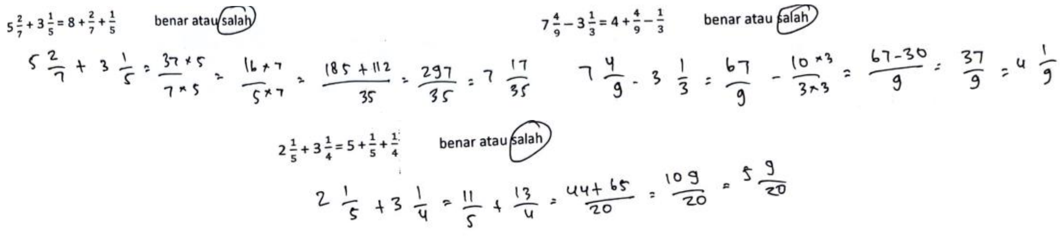


Figure 13. The answer of QB by S17

Based on the answer of S17 (Figure 13), he did the misconception option for the problem. To prove their work, he followed procedural steps. For Question 1, the answer he provided was, $7\frac{17}{35}$ whereas the correct answer should have been $8\frac{17}{35}$. The answers for Question 2 and 3 were correct, but the options selected were incorrect.

Question type C: Illustration (designed workbook)

Anjar, Haby, Citra, dan Wahu belajar kelompok. Pertama, mereka menyelesaikan secara individu $2\frac{1}{5}+3\frac{1}{4}=5+\frac{1}{5}+\frac{1}{4}$ benar atau salah. Masing-masing dari mereka sudah menyelesaikan soal tersebut dengan caranya masing-masing, lalu mereka berdiskusi hasil jawaban mereka.

Anjar:

$2\frac{1}{5}+3\frac{1}{4}=5+\frac{1}{5}+\frac{1}{4}$ adalah benar.

Pertama, hitung $2\frac{1}{5}+3\frac{1}{4}$.

$2\frac{1}{5}+3\frac{1}{4}=\frac{11}{5}+\frac{13}{4}=\frac{44}{20}+\frac{65}{20}=\frac{109}{20}=5\frac{9}{20}$

Kemudian, hitung $5+\frac{1}{5}+\frac{1}{4}=5+\frac{4}{20}+\frac{5}{20}=5\frac{9}{20}$.

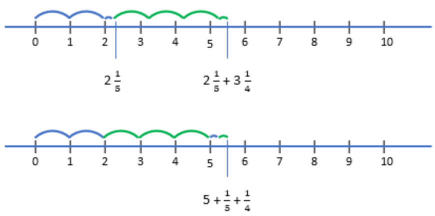
Hasil dari ruas kanan dan kiri sama, sehingga pernyataannya bernilai benar.

Haby:

$2\frac{1}{5}+3\frac{1}{4}=5+\frac{1}{5}+\frac{1}{4}$ benar. Haby menulis kembali pernyataannya. $2\frac{1}{5}+3\frac{1}{4}=2+\frac{1}{5}+3+\frac{1}{4}=2+3+\frac{1}{5}+\frac{1}{4}=5+\frac{1}{5}+\frac{1}{4}$. Sehingga, pernyataan ini bernilai benar.

Citra:

$2\frac{1}{5}+3\frac{1}{4}=5+\frac{1}{5}+\frac{1}{4}$ adalah benar. Saya menggunakan garis bilangan. Saya menggambar bagian pertama $2\frac{1}{5}+3\frac{1}{4}$.



Bagaimana menurutmu dengan yang dikerjakan oleh Anjar, Haby dan Citra? Bagaiamana pendekatan berbeda yang mereka kerjakan? Apakah pendekatan mereka sama?

Anjar, Haby, Citra, and Wahu work together. First, they individually solved the statement $2\frac{1}{5}+3\frac{1}{4}=5+\frac{1}{5}+\frac{1}{4}$ true or false. When all four students had answered the question, they compared their work.

Anjar:

$2\frac{1}{5}+3\frac{1}{4}=5+\frac{1}{5}+\frac{1}{4}$ is true.

First, I calculated $2\frac{1}{5}+3\frac{1}{4}$.

$2\frac{1}{5}+3\frac{1}{4}=\frac{11}{5}+\frac{13}{4}=\frac{44}{20}+\frac{65}{20}=\frac{109}{20}=5\frac{9}{20}$.

Next, I calculated $5+\frac{1}{5}+\frac{1}{4}=5+\frac{4}{20}+\frac{5}{20}=5\frac{9}{20}$.

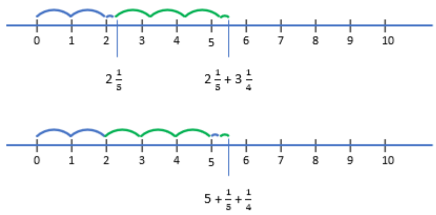
The two sides are the same, so the statement is true.

Haby:

$2\frac{1}{5}+3\frac{1}{4}=5+\frac{1}{5}+\frac{1}{4}$ is true. I rewrote the statement $2\frac{1}{5}+3\frac{1}{4}=2+\frac{1}{5}+3+\frac{1}{4}=2+3+\frac{1}{5}+\frac{1}{4}=5+\frac{1}{5}+\frac{1}{4}$. So, the statement is true.

Citra:

$2\frac{1}{5}+3\frac{1}{4}=5+\frac{1}{5}+\frac{1}{4}$ is true. I use a number line. I draw the first part $2\frac{1}{5}+3\frac{1}{4}$.

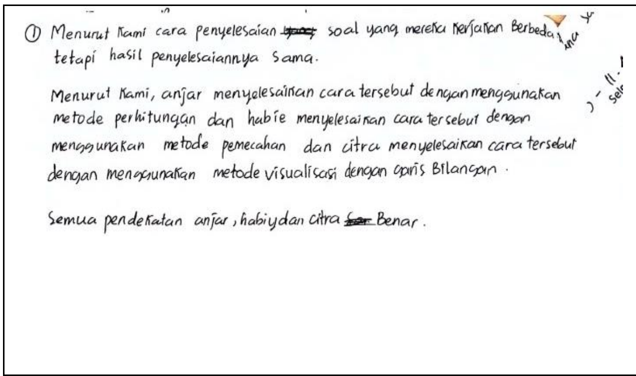


What do you think of Anjar's, Haby's and Citra's ideas? How are their approaches different, how are their approaches the same?

Figure 15. Question type C

In Question Type C, the students were given an illustration problem. The illustration provided describes a study group consisting of Anjar, Haby, Citra, and Wahu approaches. The questions can be seen in Picture 15. In this study group, the students solved the problems individually and then discussed their answers with pairs. Based on this illustration, the students were asked questions such as: *"What do you think about the work done by Anjar, Haby, and Citra? What approach did they use? Are the approaches they used the same? And is there any student who can solve the fraction problem using contextual situations like Wahu?"*

For this question, 27 students were able to complete the problem up to this stage using various methods. One of student answers can be seen in Figure 16 below:


① Menurut kami cara penyelesaian ~~yang~~ soal yang mereka kerjakan berbeda tetapi hasil penyelesaiannya sama.

Menurut kami, anjar menyelesaikan cara tersebut dengan menggunakan metode perhitungan dan habie menyelesaikan cara tersebut dengan menggunakan metode pemecahan dan citra menyelesaikan cara tersebut dengan menggunakan metode visualisasi dengan garis bilangan.

Semua pendekatan anjar, habiy dan citra ~~ber~~ benar.

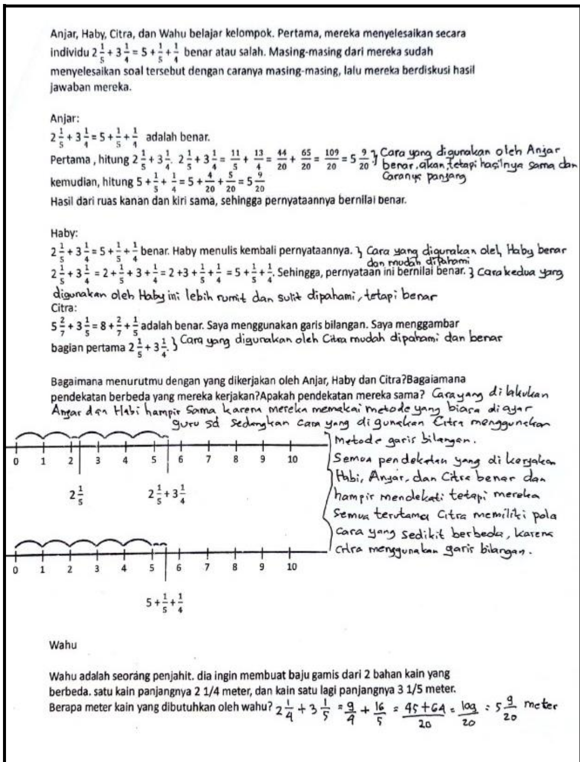
Translation

In our opinion, the methods used by Anjar, Haby, and Citra are different, but the results are the same. In our opinion, Anjar used a calculation method, and Haby used a classification method, while Citra used a visualization method with a number line. All approaches by Anjar, Haby, and Citra are correct

Figure 16. The answer of QC by S3

Based on the answer of S3 in Picture 16, she thought that the three approaches—Anjar, Haby, and Citra—used different methods to achieve the same result. Anjar used the calculation method, which most likely involved the use of numbers and formulas to get the answer, Haby used the classification of integer and fractions method, and Citra used visualization with a number line, which means she might visualize the concept of numbers in the form of a line or diagram to solve the problem. According to S3, although their methods were different, all three approaches—including the approach used by Citra—were considered correct and produced identical results. This suggests that there is more than one way to reach the correct conclusion in the context discussed. However, in the answer of S3, she did not mention about Wahu approach.

Another answer can be seen in Figure 17 below:


Anjar, Haby, Citra, dan Wahu belajar kelompok. Pertama, mereka menyelesaikan secara individu $2\frac{1}{5}+3\frac{1}{4}=5+\frac{1}{5}+\frac{1}{4}$ benar atau salah. Masing-masing dari mereka sudah menyelesaikan soal tersebut dengan caranya masing-masing, lalu mereka berdiskusi hasil jawaban mereka.

Anjar:
$2\frac{1}{5}+3\frac{1}{4}=5+\frac{1}{5}+\frac{1}{4}$ adalah benar.
Pertama, hitung $2\frac{1}{5}+3\frac{1}{4}$. $2\frac{1}{5}+3\frac{1}{4}=\frac{11}{5}+\frac{13}{4}=\frac{44}{20}+\frac{65}{20}=\frac{109}{20}=5\frac{9}{20}$ } Cara yang digunakan oleh Anjar benar, akan tetapi hasilnya sama dan caranya panjang
kemudian, hitung $5+\frac{1}{5}+\frac{1}{4}=5+\frac{4}{20}+\frac{5}{20}=5\frac{9}{20}$
Hasil dari ruas kanan dan kiri sama, sehingga pernyataannya bernilai benar.

Haby:
$2\frac{1}{5}+3\frac{1}{4}=5+\frac{1}{5}+\frac{1}{4}$ benar. Haby menulis kembali pernyataannya. } Cara yang digunakan oleh Haby benar dan mudah dipahami
$2\frac{1}{5}+3\frac{1}{4}=2+\frac{1}{5}+3+\frac{1}{4}=2+3+\frac{1}{5}+\frac{1}{4}=5+\frac{1}{5}+\frac{1}{4}$. Sehingga, pernyataan ini bernilai benar. } Cara kedua yang digunakan oleh Haby ini lebih rumit dan sulit dipahami, tetapi benar
Citra:
$5\frac{2}{7}+3\frac{1}{5}=8+\frac{2}{7}+\frac{1}{5}$ adalah benar. Saya menggunakan garis bilangan. Saya menggambar bagian pertama $2\frac{1}{5}+3\frac{1}{4}$. } Cara yang digunakan oleh Citra mudah dipahami dan benar

Bagaimana menurutmu dengan yang dikerjakan oleh Anjar, Haby dan Citra?Bagaiamana pendekatan berbeda yang mereka kerjakan?Apakah pendekatan mereka sama? Cara yang dilakukan Anjar dan Habi hampir sama karena mereka memakai metode yang biasa diajar guru sd sedangkan cara yang digunakan Citra menggunakan metode garis bilangan.

0 1 2 3 4 5 6 7 8 9 10
$2\frac{1}{5}$ $2\frac{1}{5}+3\frac{1}{4}$

Semoa pendekatan yang dikerjakan Habi, Anjar, dan Citra benar dan hampir mendekati tetapi mereka semua terutama Citra memiliki pola cara yang sedikit berbeda, karena Citra menggunakan garis bilangan.

0 1 2 3 4 5 6 7 8 9 10
$5+\frac{1}{5}+\frac{1}{4}$

Wahu

Wahu adalah seorang penjahit. dia ingin membuat baju gamis dari 2 bahan kain yang berbeda. satu kain panjangnya 2 1/4 meter, dan kain satu lagi panjangnya 3 1/5 meter.
Berapa meter kain yang dibutuhkan oleh wahu? $2\frac{1}{4}+3\frac{1}{5}=\frac{9}{4}+\frac{16}{5}=\frac{45+64}{20}=\frac{109}{20}=5\frac{9}{20}$ meter

Translate:

Anjar: The method used by Anjar is correct, but the results are the same and the method is long.

Haby: The method used by Haby is correct and easier to understand. However, in the second strategy, it is more complicated and difficult to understand, but it is correct.

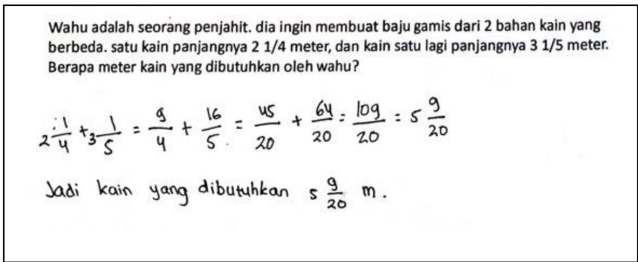
Citra: the method used by Citra is easy to understand and correct.

The methods used by Anjar and Haby are almost the same because they use methods that usually be taught by elementary school teachers, while the method used by Citra uses the number line method. All approaches used by Haby, Anjar, and Citra are correct and almost similar except for Citra has slightly different method patterns, because Citra uses number lines.

Figure 17. The answer of QC by S14

Based on Figure 17, S14 provided a detailed comparison of the different methods used by Anjar, Haby, and Citra in solving the problem, while highlighting the advantages and disadvantages of each method. According to S14, Anjar uses the correct method but requires longer steps to achieve the same result. This shows that Anjar's method may be more detailed or layered, although the end result is comparable to the others. Haby has a correct method with a more intuitive approach by making classification or directly adding integers. Meanwhile, Citra's approach may be simpler or more visual, especially because she uses the number line method. Overall, S14 stated that although the methods used by Anjar, Haby, and Citra are slightly different, they are all correct. Furthermore, she said that Anjar and Haby's methods are almost the same, because they both use an approach commonly taught in elementary schools, while Citra's method differs because it uses a number line, providing a unique solution pattern. The small differences in the pattern of these methods, especially the one used by Citra, show that there are various ways to achieve the correct result, although some approaches may be easier to understand or more complicated depending on the individual using them.

The final question in Type C aims to provide students with an understanding that fractional problems can be related to contextual situations. Thus, when they encounter fractional numbers, Wahu illustration demonstrates that these fractions are analogous to something found in everyday life. In this case, the context used is the length of fabric in meters. The question is: *"Wahu is a tailor, he wants to make a dress from 2 different fabrics. One fabric is* $2\frac{1}{4}$ *meters long, and the other fabric is* $3\frac{1}{5}$ *meters long. How many meters of fabric does Wahu need?"* Based on the answer sheets, students were able to solve this question using CLMOT strategies.


Wahu adalah seorang penjahit. dia ingin membuat baju gamis dari 2 bahan kain yang berbeda. satu kain panjangnya 2 1/4 meter, dan kain satu lagi panjangnya 3 1/5 meter. Berapa meter kain yang dibutuhkan oleh wahu?

$2\frac{1}{4} + 3\frac{1}{5} = \frac{9}{4} + \frac{16}{5} = \frac{45}{20} + \frac{64}{20} = \frac{109}{20} = 5\frac{9}{20}$

Jadi kain yang dibutuhkan $5\frac{9}{20}$ m.

Wahu is a tailor, he wants to make a dress from 2 different fabrics. One fabric is $2\frac{1}{4}$ *meters long, and the other fabric is* $3\frac{1}{5}$ *meters long. How many meters of fabric does Wahu need?*

Figure 18. The answer of QC by S7

Based on S7 answer in Figure 18, students used a procedural approach, starting from the stage of converting mixed numbers to improper fractions up to the stage of turning the fraction into a mixed number (CLMOT).

Meanwhile, some students reached the stage of decimal results such as follows.

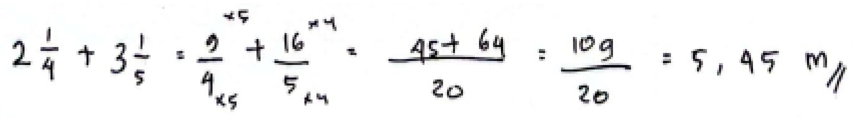

$2\frac{1}{4} + 3\frac{1}{5} = \frac{9^{\times 5}}{4_{\times 5}} + \frac{16^{\times 4}}{5_{\times 4}} = \frac{45+64}{20} = \frac{109}{20} = 5,45$ m

Figure 19. The answer of QC by S27

Question type D: True/false based on the illustration analysis

Type D questions are similar to Type B questions in that they require students to analyze whether statements are true or false. However, these questions are based on the illustrations from the approaches of Anjar, Haby, Citra, and Wahu. In other words, in this question, students were asked to analyze fraction calculation problems using the approach of Anjar, Haby, Citra, and Wahu. This aims to help students better understand the differences among the four approaches, which will, in turn, assist them in grasping the meaning of fraction operations without relying solely on procedural methods that they may not fully understand.

Question type D number 1

Selesaikan soal berikut menggunakan pendekatan Anjar, Haby, Citra dan Wahu. $3\frac{1}{3}-2\frac{1}{4}=1+\frac{1}{3}-\frac{1}{4}$ benar atau salah		Solve these two problems using the three approaches from Anjar, Haby, Citra and Wahu. $3\frac{1}{3}-2\frac{1}{4}=1+\frac{1}{3}-\frac{1}{4}$ true or false	
Pendekatan Anjar		Anjar's Approach	
Pendekatan Haby		Haby's Approach	
Pendekatan Citra		Citra's Approach	
Pendekatan Wahu		Wahu's Approach	

Figure 20. Question type D number 1

For the first question, most students were able to complete the problem using the approaches applied by Anjar and Haby. They chose the "true" option using procedural strategy. However, they skipped the approach of Citra. Furthermore, for the Wahu approach, they did not apply a contextual situation. Instead, they used the same strategy as Anjar approach. The example of the students' answer is shown below in Figure 21.

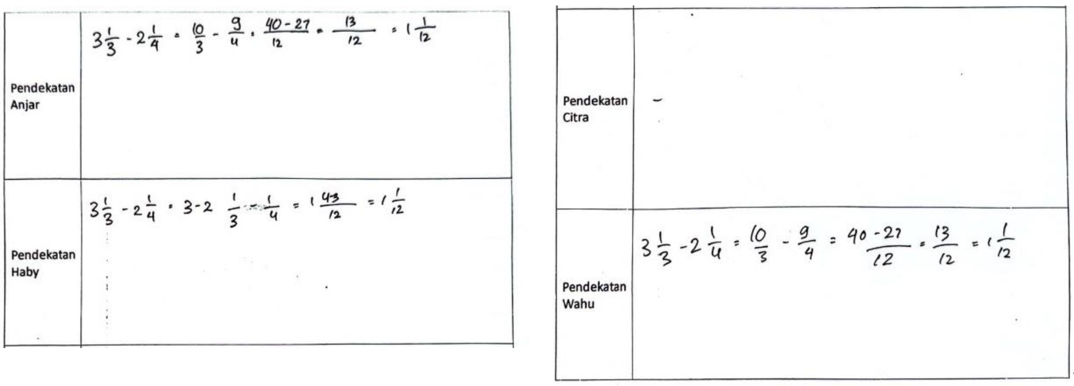


Figure 21. The answer of QD1 by S6

Moreover, there was a few students who provide the complete answer including the approach of Citra such in the following:

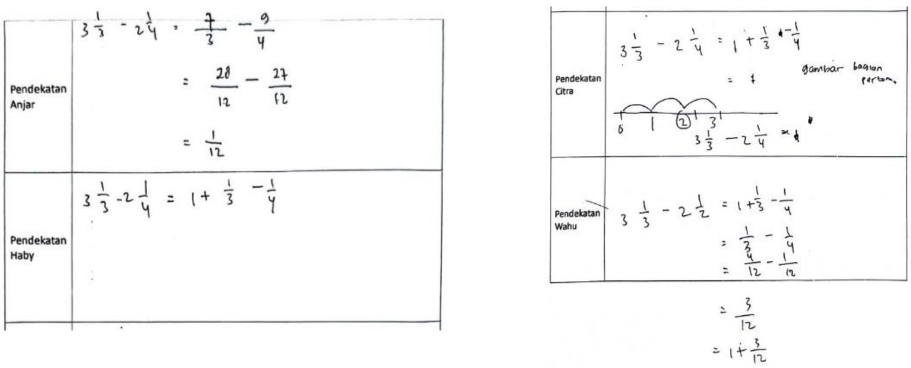


Figure 23. The answer of QD1 by S9

Based on the answer from S9 in Figure 23, which is similar to S6, she used the procedural strategy (CLMOT) for both the Anjar and Wahu approaches, however she missed the calculation, and she got incorrect results. For Haby approach, she was able to apply the classification of integer and fractional numbers but she did not continue her work. For Citra approach, she showed a number line with points represented by the numbers from 0 to 3. There were several arcs connecting the points on the number line, starting from 0 on the left. Then, there were three arcs that indicate the addition of numbers one by one, from 0 to 1, 1 to 2, and 2 to 3. The number 2 was circled, possibly to mark a specific point on the number line. Below the number line, there was $3\frac{1}{3} - 2\frac{1}{4}$ which represents the operation being calculated or explained through the number line above it. However, she did not come up to the final answer, and it seems that the decomposed numbers were used to explain fraction arithmetic operations with the help of the number line.

Question type D number 2

$6\frac{1}{15} - 3\frac{1}{5} = 3\frac{4}{15}$ Benar atau salah		$6\frac{1}{15} - 3\frac{1}{5} = 3\frac{4}{15}$ true or false	
Pendekatan Anjar		Anjar's Approach	
Pendekatan Haby		Haby's Approach	
Pendekatan Citra		Citra's Approach	
Pendekatan Wahu		Wahu's Approach	

Figure 24. Question type D number 2

For question number 2 of Type D, it is almost similar to question number 1. However, in question 2, the two numbers are mixed fractions consisting of both integers and fractions that are to be operated on. Similar to the first question, for the second question, most students were able to solve the problem using the approaches applied by Anjar and Haby. They chose the "true" option using procedural strategy. However, they skipped the approach of Citra. Furthermore, for the Wahu approach, they did not apply a contextual situation. Instead, they used the same strategy as Anjar approach. The example of the students' answer is shown below.

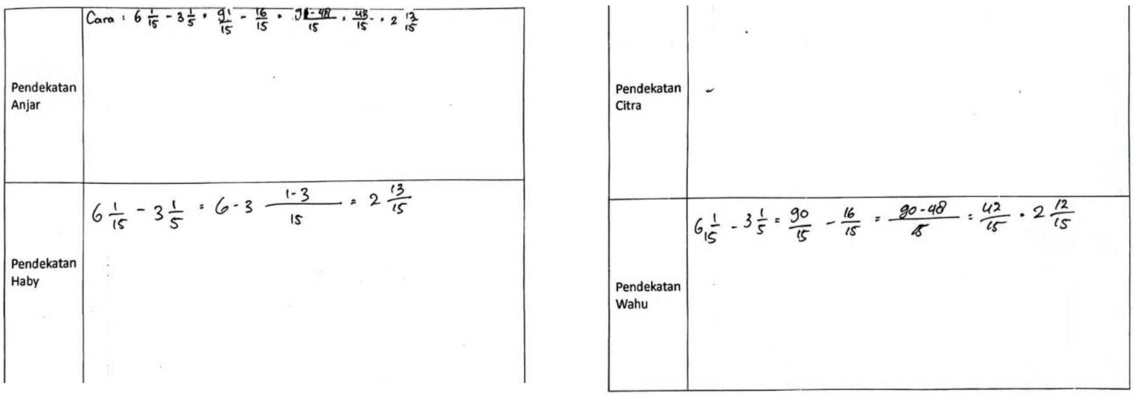


Figure 25. The answer of QD2 by S6

Meanwhile, when using Citra's number line approach, no student provided an answer to question 2. When we interviewed them, they said that using the number line was not very familiar to them, and they still did not understand it.

Question type E: Mixed fraction based on the illustration analysis

In this problem, the students were given formal fraction calculations similar to those in Type A. At this stage, it was expected that they would have been inspired by the designed workbook including illustrations of the approaches of Anjar, Haby, Citra, and Wahu. Therefore, the aim of this question was to evaluate whether the workbook successfully encouraged students to shift their thinking, moving away from merely applying procedural solutions to a deeper understanding of the meaning behind fraction operations.

Berikan ide dari Anjar, Haby dan Citra bagaiamana kamu sekarang menyelesaikan soal-soal berikut.	Given the ideas of Anjar, Haby, and Citra how would you now solve the following problems.
$2\frac{11}{14} - 1\frac{1}{7} =$	$2\frac{11}{14} - 1\frac{1}{7} =$
$7\frac{2}{11} - \frac{1}{2} =$	$7\frac{2}{11} - \frac{1}{2} =$
$4\frac{11}{12} - 2\frac{5}{8} =$	$4\frac{11}{12} - 2\frac{5}{8} =$

Figure 26. Question type E number 1-3

In Question Type E, there are 3 questions consisting of the subtraction of mixed fractions from mixed fractions, and the subtraction of ordinary fractions from mixed fractions.

Question type E number 1

For the first question, it includes the subtraction of mixed fractions from mixed fractions. 20 students were able to solve the problem using Anjar's approach and 7 people were able to solve the problem using Haby's approach. One of example of student’s answers can be seen as follows:

1. Anjar Approach

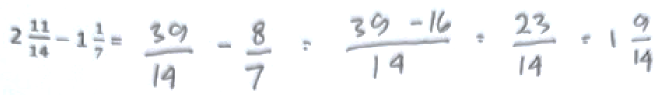


Figure 27. The answer of QE1 by S3

Based on the Figure 27 above, it can be indicated that Anjar approach contains procedural approach contains CLMOT. The steps used included convert the mixed fractions become improper fractions, and then equate the denominators by finding the LCM which results in 14. Then, he multiplied the numerator with the same number needed to get the common denominator and after that he subtracted the numerators, so that the result is $\frac{23}{14}$. Finally, he turned the ordinary fraction into a mixed fraction, so that the numbers obtained can be simpler and the final result obtained is $1\frac{9}{14}$. The reasons S3 chose the Anjar approach can be seen in Figure 28 below.

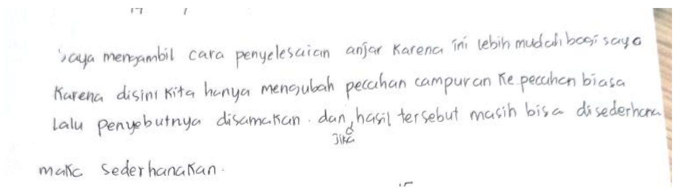
 saya mengambil cara penyelesaian anjar karena ini lebih mudah bagi saya karena disini kita hanya mengubah pecahan campuran ke pecahan biasa lalu penyebutnya disamakan dan jika hasil tersebut masih bisa disederhanakan maka sederhanakan.	***Translate*** *I took Anjar approach because it is easier for me, and it is because here we only change the mixed fractions to ordinary fractions, then the two denominators are made the same and then simplified.*

Figure 28. The reason of QE1 by S3

The statement in Figure 28 described the reason why S3 chose Anjar's approach because he thinks that it is easier to understand and follow. This is in line with the interview result as follows:

<u>Dialogue 1</u>:

R: There are three approaches: Citra, Haby, and Anjar. How do you see them based on the discussion here?

S3: According to our group, each of them used a different approach. Citra used the number line approach, Haby rewrote the statement, and Anjar's approach was more elaborated.

According to Dialogue 1, S3 considered Anjar approach to be more convenient for solving problems because it is more elaborated.

2. Haby approach

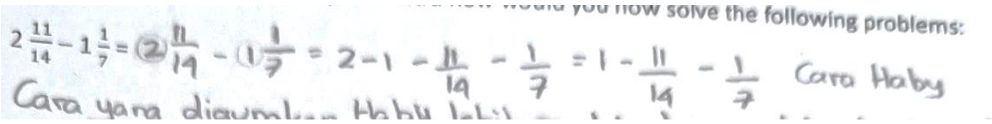


Figure 29. The answer of QE1 by S15

Based on the Figure 29 above, S15 used Haby approach by subtracting the integers, namely 2 - 1 to 1. Then, after obtaining the results of the subtraction, the results obtained are 1 - $\frac{11}{14}$ - $\frac{1}{17}$. The reason S15 used Haby approach can be seen in Figure 30 below.

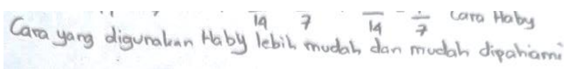
 Cara yang digunakan Haby lebih mudah dan mudah dipahami	***Translation*** *The method used by Haby is easier and more understandable*

Figure 30. The reason of QE1 by S15

The reason of S15 in Figure 30 stated that the method used by Haby is considered easier and more understandable. This means that the steps taken in Haby approach were arranged in a clear and simple way, so that students who used it can follow and understand the process better than other approaches. This approach may be more intuitive, direct, or use aids that make it easier to understand more complex concepts. To further ensure students' understanding of the various approaches used, the researcher asked one of the groups.

R: Could you share information regarding the approaches used?
S18: I used two approaches: the first Haby's approach, the second Anjar's approach. The first one is easier for something like $2\frac{1}{4}$ minus $1\frac{1}{2}$; it's easier to use Haby's approach.
R: What is Haby's approach?
S18: Haby's approach starts with the front, like 2 minus 1.
R: What is the number in front? $2\frac{1}{4}$, right? What is 2? And what is $\frac{1}{4}$?
S23: Numerator and denominator.
R: What is the term for those numbers?
Students: Fraction.
R: What is 2?
Students: A whole number.
R: There is a fraction, $\frac{1}{4}$. What about 2? If it is not a fraction, what is it?
Students: Integer.
R: Integer, right? So how does Haby's approach work?
S18: Add the integer first, then the fractions. For question 2, 3, we used Anjar's approach because it's easier.
R: So, there is a difference between question 1 and the others?
S18: Yes, there is. For question 2 and 3, I used Anjar's approach, but for question 1, I used Haby's approach, depending on the question.

Question type E number 2

In the second question, it includes the subtraction of ordinary fractions from mixed fractions. In question number 2 type E, no one chose an approach other than Anjar. The example of students' answer can be seen as follows:

1. Anjar Approach

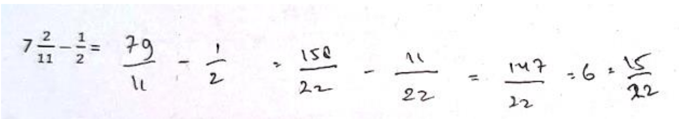


Figure 31. The answer of QE2 by S14

Based on the Figure 31 above, it can be indicated that S2 has understood the concept of subtracting fractions with different denominators. He used Anjar approach or procedural steps using CLMOT strategies. The steps taken include equating the denominators of the fractions by converting the mixed fractions to ordinary fractions, finding the LCM, which produces the number 22. After that, the student subtracted the numerators, resulting in $\frac{147}{22}$. Then, he changes the improper fraction into a mixed fraction to simplify the result, which finally becomes $6\frac{15}{22}$. The reason S14 chose Anjar approach can be seen in Figure 32 below.

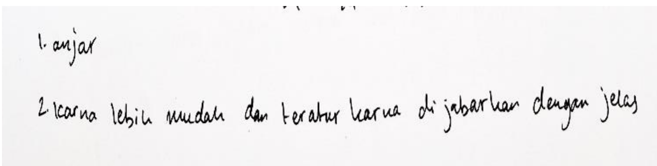
	***Translation*** ***Anjar*** *because it is easier and more organized, because it is explained clearly*

Figure 32. The answer of QE2 by S14

The reason of S14 in Figure 32 highlights that Anjar's approach is considered easier to follow and more organized because it is delivered with clear explanations.

Question type E number 3

The third question includes the subtraction of mixed fractions from mixed fractions. Similar to the first question in type E.

1. Anjar Approach

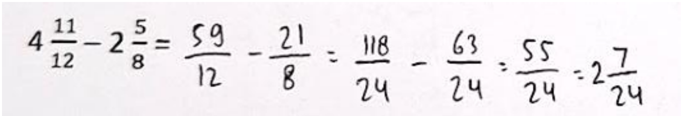


Figure 33. The answer of QE3 by S13

Based on the answer in Figure 33, it can be indicated that S3 used Anjar approach or procedural steps (CLMOT). The steps taken included convert the mixed fractions to ordinary fractions, equated the denominators of the fractions by finding the LCM, which resulted in the number 24. After that, S13 subtracted the numerators, resulting in $\frac{55}{24}$ from the subtraction of $\frac{118}{24} - \frac{63}{24}$. Then, he turned the improper fraction to a mixed fraction to simplify the result, which finally became $2\frac{7}{24}$. The reason S13 chose the Anjar approach can be seen in Figure 34 below.

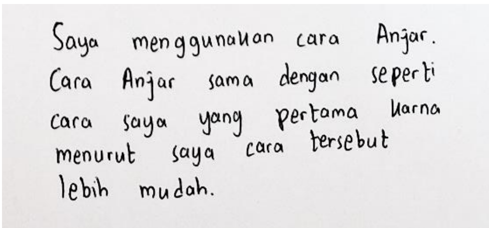
	***Translation*** *I used Anjar method because it is the same as my first method because I think it is easier.*

Figure 34. The reason of QE3 by S13

Figure 34 explains that S3 chose Anjar approach because she felt that it was easier. This is in line with the interview result as follows:

<u>Dialogue 3</u>

R: Why do you use Anjar method?
S13: Because Anjar's method is usually what we use, it's easier to do.
R: How about number 1?
S13: Yes, it's the same.

2. Haby Approach

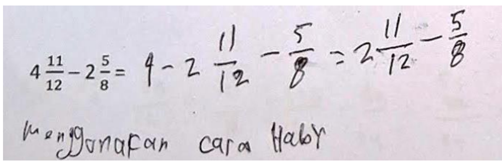
	***Translation*** *using Haby method*

Figure 35. The answer of QE3 by S10

Based on the answer in Figure 35, S10 used Haby approach by subtracting the integers, namely 4 - 2 to 2. Then, after obtaining the results of the subtraction, the results obtained are 2 $\frac{11}{12}$ - $\frac{5}{8}$. However, S15 did not provide detail reason for why he used Haby approach. He simply stated that he prefers to use Haby approach.

Discussion

This study used data obtained from a series of questions given to Pre-service primary School Teachers designed in a workbook. The workbook consisted of five questions that asked students to solve the calculation of mixed fractions using their respective approaches or methods. In general, the steps taken by pre-service teachers who became the target research include: 1) Convert the mixed numbers to improper fractions (C); 2) Find the Least Common Multiple (LCM) of the fractions (L); 3) Multiply the numerator with the same number needed to get the common denominator (M); 4) Operate numerators (O); 5) Turn the fraction into a mixed number (T); 6) Simplify the new fraction (S).

However, because the pre-service teachers have been given treatment with a workbook, they are free to choose a procedural or conceptual approach illustrated by Anjar, Haby, Citra, or Wahu approaches. The aim of this question was to evaluate whether the workbook successfully encouraged students to shift their thinking, moving away from merely applying procedural solutions to a deeper understanding of the meaning behind fraction operations. Based on the students' answer sheets, an analysis was carried out, which included the strategies or steps used to solve the fraction problems. Pre-service teachers were expected to be able to solve the problems more effectively and accurately.

The results of this study indicate that emphasizing word problems in fractions through the design of contexts with various types of fraction concepts and building a strong understanding of fractional numbers can help pre-service teachers reduce misunderstandings and gain a deeper comprehension of fraction operations. It is beneficial to introduce a diagram or other representation to establish a connection between the context and the mathematics. Our observations indicate that context can result in meaningful learning when pre-service teachers participate actively in the conversation by posing questions to elucidate, justify, and explain their thinking. A fractional problem was administered to assess pre-service teachers' mathematical proficiency in teaching fractions. The test's primary purpose was to ascertain their level of subject knowledge regarding fractions. The test had several components: Participants had to look up questions, respond to them, and provide justifications for their responses. Their content knowledge was connected to each problem's solution and the justifications for their instructional expertise.

According to Anderson in Duodu et al. (2019), pre-service teachers must gain the necessary knowledge and ability to teach mathematics through problem-solving. Moreover, research indicates that prospective teachers frequently struggle to deeply understand how to promote mathematical reasoning and assist students in navigating challenging problem-solving situations (Masingila et al., 2017). Nonetheless, the differences demonstrated that many pre-service teachers struggle with understanding fractions. Making pre-service teachers' instructors aware of their understanding of topics will be improved by exposure to various fractional models (Duodu et al., 2019). Nevertheless, many prospective teachers struggle to effectively model fraction operations, suggesting the necessity for additional training and enhancement in their teaching methods (Lee & Lee, 2022). In line with this, the test utilized various fraction models, including the illustration using the approach of Anjar, Haby, Citra, and Wahu.

This study is essential for understanding why pre-service teachers struggle with fractions. However, the results highlight the significance of opportunities for professional development for teachers, particularly those in primary school education, to support their conceptual growth in fraction calculation. The study's findings support past research that indicates teachers' comprehension of fraction operations needs to be improved (Gencturk, 2021) and that students continue to make errors and hold misconceptions about fractions, particularly when performing fraction calculations (Ratnasari, 2018). It is widely recognized that students need multiple opportunities to link various conceptual frameworks and visual models of rational numbers to fully grasp fraction concepts (Wilkie & Roche, 2023). However, the findings go beyond these observations by shedding light on teachers' difficulties. A significant outcome of this study is that even for the comparatively simpler method (adding fractions), only some of the pre-service teachers gave justifications that focused on the operation's mathematical foundations. The outcomes of their problem-solving skills demonstrated how little pre-service teachers knew about fractions in terms of conceptual and pedagogy. According to the study, pre-service teachers are more likely to have the first level of problem-solving skills—understanding the problem—than the subsequent levels. It indicates that the pre-service teachers lack the necessary expertise.

In applied teaching, fraction concepts are frequently taught through procedures and memorization rather than allowing students to develop their own understanding. (Getenet and Callingham, 2017). When teaching fractions to students in small groups, manipulatives are used along with conversation. The students' explicit encouragement of asking allowed them to draw on more information, like knowledge of making "tables" and repetitive addition, and connect this to fractional comprehension.

To teach mathematics to others with profound comprehension, one must possess high levels of conceptual understanding of basic mathematics (Zerpa et al., 2009). This principle is supported by research emphasizing the importance of teachers' proficiency in mathematics. Teachers need strong conceptual foundations to help students build meaningful connections between topics and apply mathematical reasoning inside and outside the classroom (Walle, 2001). As a result, this research indicates that it is essential to implement several measures for pre-service teachers to equip them with these problem-solving techniques. The study found that pre-service teachers had differing perspectives on problem-solving, especially when it comes to whether it is a "method of teaching" or a "means of finding a solution." pre-service teachers, who will shortly be implementing problem-solving techniques in fundamental mathematics classrooms, create issue differentiating solutions that ought to be viewed as a national priority because of instructors' classrooms. Their conceptions guide their practices.

The participants felt that comprehending mathematics is essential and effective instruction should always support this. While memorization, practice, and hands-on experience are not seen as right or wrong, they are essential to comprehending mathematics. Effective teachers use these strategies to make learning understandable. Researchers have long noted that students' misconceptions about fractions hinder their ability to manipulate them effectively. The common view of fractions as parts of a whole must be improved to foster a comprehensive understanding of fractions. This limited perspective restricts students' understanding of improper fractions (Brown, 2016).

The study emphasizes how critical it is to comprehend the viewpoints, experiences, and beliefs that influence mathematics teachers' methods of instruction. It also highlights the significance of continuous professional development in helping educators gain a deeper comprehension of mathematics as a source of applicable knowledge. Furthermore, this research emphasizes how crucial it is to have a nurturing learning atmosphere that inspires children to form relationships between ideas in mathematics and actual circumstances. This study offers insightful information about the intricate interactions among instructors' knowledge, beliefs, and social circumstances to shape how they approach teaching and learning numerical methods (Kasa et al., 2024). According to the instructors under study, mathematics is a dynamic and coherent body of knowledge that has been honed through the solution of practical problems and is thus helpful in resolving practical problems. They understand that mathematics is not an abstract topic and that it is essential to handle humanity's most important issues. As a result, they contend that mastering mathematics is an essential learning goal and that educators must use different strategies to help their pupils grasp mathematics.

Based on this study, the approaches used by pre-service primary school teachers refer to Anjar and Haby's approaches. It indicates that most pre-service teachers in this study still need to fully understand the meaning behind using the procedural methods they were accustomed to during primary school. However, using the designed workbook provided in this study had a noticeable, though insignificant, impact. Some pre-service teachers shifted their thinking from Anjar's procedural approach to Haby's conceptual approach. Nevertheless, by the end of the study, when presented with questions, none of the pre-service teachers chose Citra's approach, which involves using a number line. It suggests they need to become more familiar with using number lines, even though they acknowledge it as new knowledge.

Moreover, the expected contextual approach needs to be revised, as none of them ultimately connected the fractions to real-life situations, as demonstrated in Wahu's illustration. It is essential to provide a simple introduction to contextual issues while concluding with a higher numerical method (Widjaja, 2013). However, it is acknowledged that the scope of this study is restricted to analyzing the written assignments and brief interviews with the pre-service teachers; a more thorough analysis may be produced if pre-service teachers were observed and tracked for an extended duration. Examining the evolution of mathematical comprehension over time is necessary to document the students' growing process comprehension. Therefore, it is crucial to look at the development of mathematics by pre-service teachers throughout time and in the social environment in which learning takes place (Nillas, 2003).

Conclusion

Supporting pre-service primary school teachers in understanding the meaning of a mathematical concept like fractions remains highly challenging. This issue suggests that although pre-service teachers have studied fractions, they still need to understand the fundamental concepts of fractions. Based on these findings, it is recommended that when pre-service primary school teachers learn about fractions, their understanding of the meaning of fractions should be effectively addressed through problems that challenge this contextual situation. While the findings can show a range of answers from diverse pre-service teachers, there are limitations related to the participants' responses through the problems and questions provided by the researchers in the designed workbook. Since the results show that problems with some unfamiliar about various approaches, such as using number lines and contextual situations, further research should provide additional details on how these issues are addressed in larger groups of participants over a longer period, with a more elaborate teaching and learning design, it would help develop knowledge for educators in teaching fractions, particularly in stimulating students' mathematical problem-solving skills.

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Conflicts of Interest

The authors declare no conflict of interest regarding the publication of this manuscript. In addition, the authors have completed the ethical issues, including plagiarism, misconduct and data fabrication or falsification, double publication or submission and redundancies.

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Author Contributions

Puri Pramudiani: Conceptualization, Writing - Original Draft, Editing and Visualization; **Fitri Alyani:** Analyzing, Literature Review, Editing and Visualization; **Maarten Dolk:** Review and Editing, Formal Analysis, Validation and Supervision; **Wanty Widjaja:** Review and Editing, Methodology, Validation and Supervision.

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