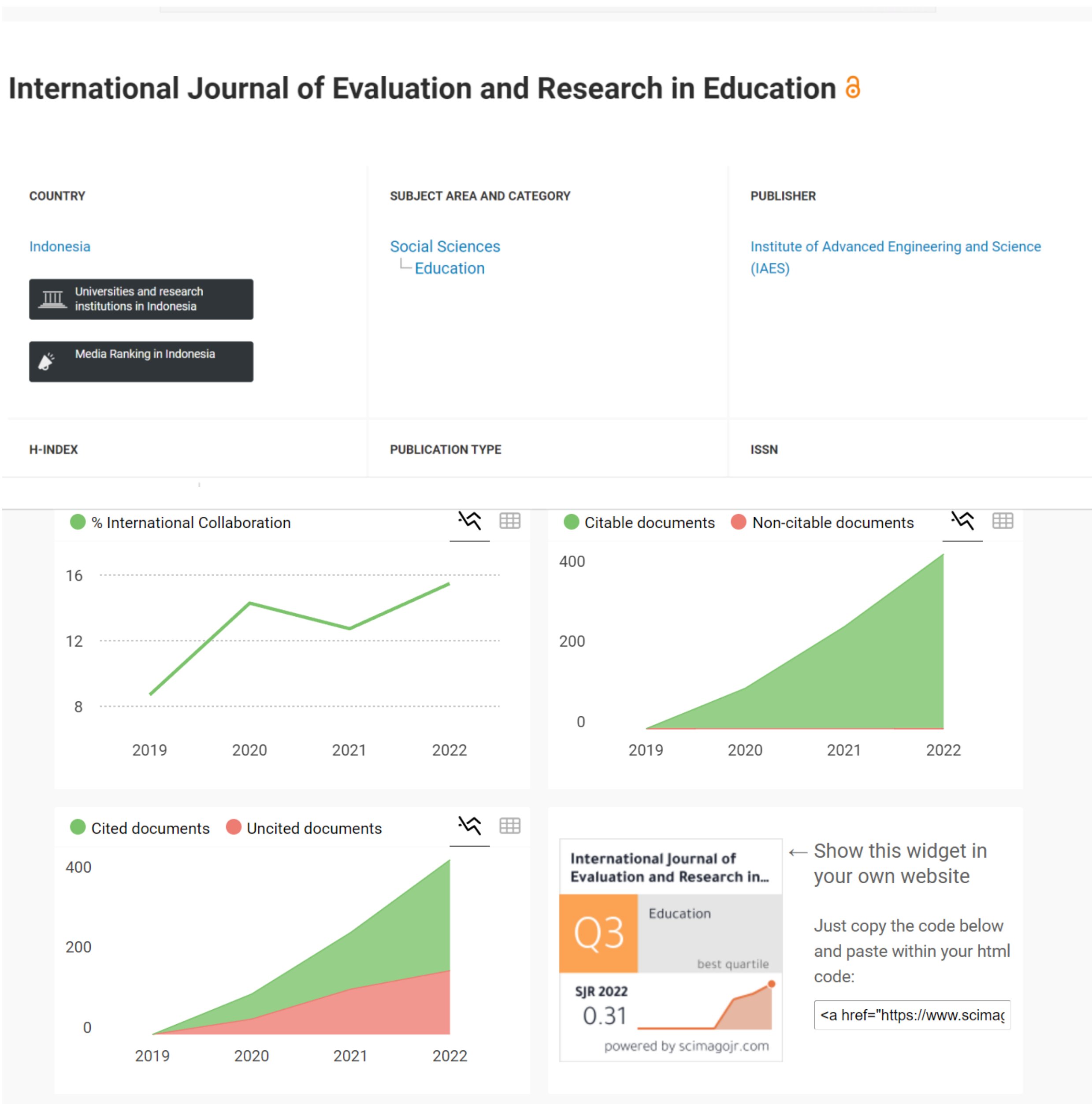
Bukti Reviewer Pada International Journal of Evaluation and Research in Education

Link: https://ijere.iaescore.com/index.php/IJERE/reviewer/submission/51979

Id: samsul

Pasword: r9a8Eq93



Intelektual Pustaka Media Utama

iaes Institute of Advanced Engineering and Science

Scopus® SJR SCImago Journal & Country Rank
CiteScore 2022 : 2.1
SNIP 2022 : 0.821
SJR 2022 : 0.308

CERTIFICATE

No: 28217/IJERE/R1/VIII/2023

International Journal of Evaluation and Research in Education

is hereby awarding this certificate to

Samsul Maarif

in recognition of his/her contribution as **Reviewer** on paper ID:

28217

in this scientific journal

Yogyakarta, 15 August 2023

ISSN 2252-8822
http://ijere.iaescore.com

Prof. Dr. Yeo Kee Jiar
Editor in Chief

Gmail

Search in mail

Active

Compose

Mail 99+

Chat

Spaces

Meet

Inbox 3,497
Starred
Snoozed
Sent
Drafts 35
More

Labels +

3 of 6,942

[IJERE] Article Review Request External Inbox ×

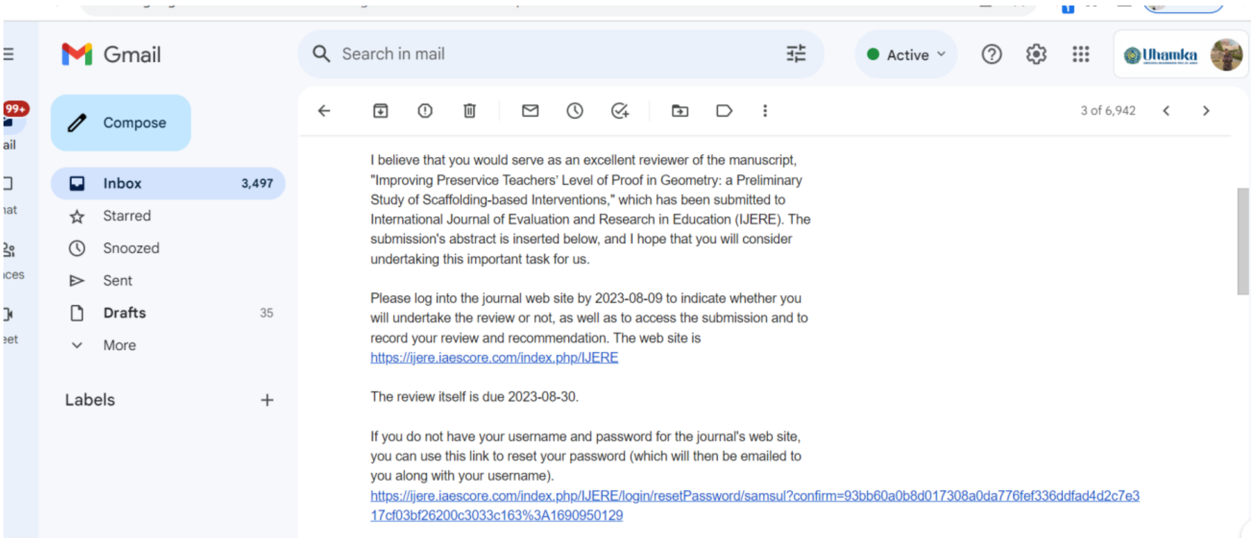
Dr. Lina Handayani ijere@iaesjournal.com via kirimdata.id 9:23 AM (1 hour ago)
to me

The following message is being delivered on behalf of International Journal of Evaluation and Research in Education (IJERE).

Samsul Maarif:

I believe that you would serve as an excellent reviewer of the manuscript, "Improving Preservice Teachers' Level of Proof in Geometry: a Preliminary Study of Scaffolding-based Interventions," which has been submitted to International Journal of Evaluation and Research in Education (IJERE). The submission's abstract is inserted below, and I hope that you will consider undertaking this important task for us.

Please log into the journal web site by 2023-08-09 to indicate whether you will undertake the review or not, as well as to access the submission and to record your review and recommendation. The web site is

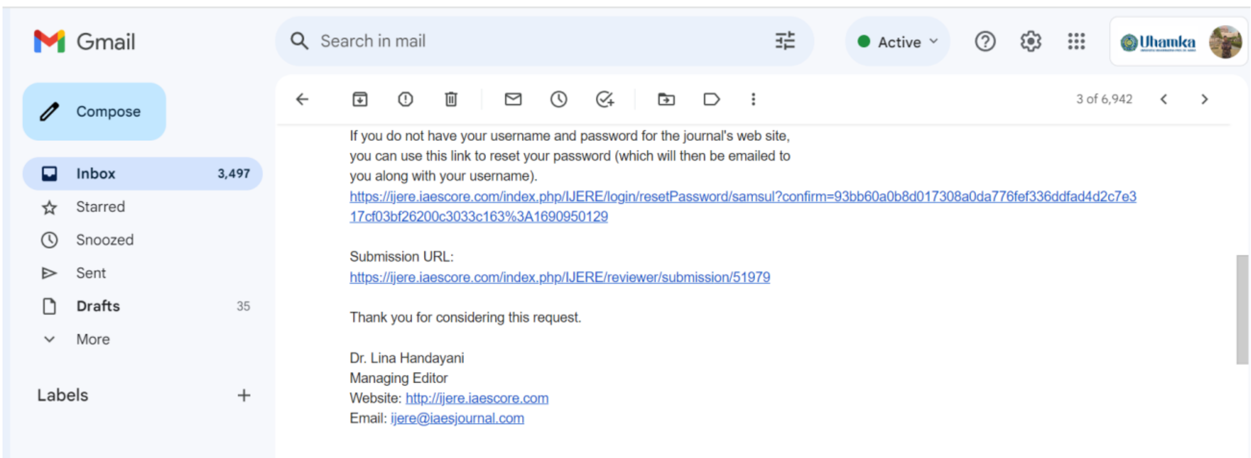


I believe that you would serve as an excellent reviewer of the manuscript, "Improving Preservice Teachers' Level of Proof in Geometry: a Preliminary Study of Scaffolding-based Interventions," which has been submitted to International Journal of Evaluation and Research in Education (IJERE). The submission's abstract is inserted below, and I hope that you will consider undertaking this important task for us.

Please log into the journal web site by 2023-08-09 to indicate whether you will undertake the review or not, as well as to access the submission and to record your review and recommendation. The web site is https://ijere.iaescore.com/index.php/IJERE

The review itself is due 2023-08-30.

If you do not have your username and password for the journal's web site, you can use this link to reset your password (which will then be emailed to you along with your username).
https://ijere.iaescore.com/index.php/IJERE/login/resetPassword/samsul?confirm=93bb60a0b8d017308a0da776fef336ddfad4d2c7e317cf03bf26200c3033c163%3A1690950129



If you do not have your username and password for the journal's web site, you can use this link to reset your password (which will then be emailed to you along with your username).
https://ijere.iaescore.com/index.php/IJERE/login/resetPassword/samsul?confirm=93bb60a0b8d017308a0da776fef336ddfad4d2c7e317cf03bf26200c3033c163%3A1690950129

Submission URL:
https://ijere.iaescore.com/index.php/IJERE/reviewer/submission/51979

Thank you for considering this request.

Dr. Lina Handayani
Managing Editor
Website: http://ijere.iaescore.com
Email: ijere@iaesjournal.com

http://ijere.iaescore.com | ISSN 2252-8822, e-ISSN 2620-5440

International Journal of Evaluation and Research In Education (IJERE)

CiteScore 2022 2.1 Scopus | Q3 | SJR 2022 0.308 | SNIP 2022 0.821 | Intelektual Pustaka Media Utama

#28217 Review

Submission To Be Reviewed

Title	Improving Preservice Teachers' Level of Proof in Geometry: a Preliminary Study of Scaffolding-based Interventions
Journal Section	General Education Concepts
Abstract	This is a preliminary study of design research that investigates preservice mathematics teachers' proof level and the possible task of scaffolding-based interventions in proving the triangle theorem. The research subjects consisted of 58 second-semester mathematics education students at Universitas Negeri Surabaya, Indonesia. This research is descriptive using quantitative and qualitative approaches. Data collection uses a test to determine the level of proof of prospective mathematics teachers based on Miyazaki's classification. This method classifies four levels in constructing a proof, mainly Proof A, Proof B (deductive), Proof C, and Proof D (inductive). The results showed that there were 38% of students' answers in constructing of proof with level Proof A, 5% of students' answers in constructing of proof with level Proof B, 15% of students' answers in constructing of proof with level Proof C, and the remaining 42% of students' answers in constructing of proof with level Proof D. Furthermore, the scaffolding-based intervention task refers to the preservice teacher's difficulties in proving the triangle theorem, including a lack of understanding of concepts, not understanding language and mathematical notation and difficulties in starting proofs..
Submission Editor	Jonathan deHaan, Ph.D. (Review)

USER

You are logged in as...
samsul

- My Profile
- Log Out

CITATION ANALYSIS

- Google Scholar
- Scholar Metrics
- Scinapse
- Scopus
- ERIC
- Web of Science

QUICK LINKS

- Author Guideline
- Editorial Boards
- Online Submissions
- Abstracting and Indexing
- Publication Ethics
- Visitor Statistics
- Contact Us

Review Schedule

Editor's Request	2023-08-02
Your Response	2023-08-02
Review Submitted	2023-08-02
Review Due	2023-08-30

Review Steps

1. Notify the submission's editor as to whether you will undertake the review.
   Response Accepted

2. Click on file names to download and review (on screen or by printing) the files associated with this submission.
   Submission Manuscript 28217-57208-1-RV.DOCX 2023-08-01
   Supplementary File(s) None

3. Click on icon to fill in the review form.
   Review Form

4. In addition, you can upload files for the editor and/or author to consult.
   Uploaded files 28217-57217-1-RV.DOCX 2023-08-02

5. Select a recommendation and submit the review to complete the process. You must enter a review or upload a file before selecting a recommendation.
   Recommendation **Revisions Required** 2023-08-02

JOURNAL CONTENT

Search

Search Scope
All
Search

Browse
- By Issue
- By Author
- By Title

INFORMATION

- For Readers
- For Authors
- For Librarians

Search in mail

Active

4 of 6,948

[IJERE] Article Review Acknowledgement

External | Inbox

Dr. Lina Handayani ijere@iaesjournal.com via kirimdata.id

to me

Aug 3, 2023, 5:17 PM (19 hours ago)

The following message is being delivered on behalf of International Journal of Evaluation and Research in Education (IJERE).

Samsul Maarif:

Thank you for completing the review of the submission, "Improving Preservice Teachers' Level of Proof in Geometry: a Preliminary Study of Scaffolding-based Interventions," for International Journal of Evaluation and Research in Education (IJERE). We appreciate your contribution to the quality of the work that we publish.

Dr. Lina Handayani
Universitas Ahmad Dahlan
Phone +62274379418
Fax +62274381523
linafkm@gmail.com

International Journal of Evaluation and Research in Education (IJERE)

ARTIKEL HASIL REVIEW

Improving Preservice Teachers' Level of Proof in Geometry: a Preliminary Study of Scaffolding-based Interventions

4

Article Info

Article history:

Received mm dd, yyyy
Revised mm dd, yyyy
Accepted mm dd, yyyy

Keywords:

Level of Proof
Scaffolding
Geometry

ABSTRACT (10 PT)

This is a preliminary study of design research that investigates preservice mathematics teachers' proof level and the possible task of scaffolding-based interventions in proving the triangle theorem. The research subjects consisted of 58 second-semester mathematics education students at Universitas Negeri Surabaya, Indonesia. This research is descriptive using quantitative and qualitative approaches. Data collection uses a test to determine the level of proof of prospective mathematics teachers based on Miyazaki's classification. This method classifies four levels in constructing a proof, mainly Proof A, Proof B (deductive), Proof C, and Proof D (inductive). The results showed that there were 38% of students' answers in constructing of proof with level Proof A, 5% of students' answers in constructing of proof with level Proof B, 15% of students' answers in constructing of proof with level Proof C, and the remaining 42% of students' answers in constructing of proof with level Proof D. Furthermore, the scaffolding-based intervention task refers to the preservice teacher's difficulties in proving the triangle theorem, including a lack of understanding of concepts, not understanding language and mathematical notation and difficulties in starting proofs..

Commented [L1]: The abstract part at least includes the introduction, method result, and conclusion. The methodology section is not yet clear according to the existing methods in the contents of the article. In this section it is written that this research uses a qualitative descriptive approach. In my opinion, that is not a methodology, because every research will definitely describe the data.

Commented [L2]: Please review this conclusion. I doubt if the teacher-to-be doesn't understand the proof of the sum of the angles in triangles. Maybe it can be analyzed from another perspective. For example: how does the teacher see the evidence as justification or wordless evidence and so on

Corresponding Author:

Sugi Hartono
Department of Mathematics Education, Pascasarjana, Universitas Negeri Surabaya, Surabaya, Indonesia
Email: sugi.20006@mhs.unesa.ac.id

1. INTRODUCTION

Proof is at the heart of mathematical thinking and deductive reasoning [1]. Hernadi [2] explains that proof is a series of logical arguments that explain the truth of a statement. Mingus & Grassl [3] define proof as a collection of statements that are true and linked together in a logical way that serve as arguments to convince other of the truth of mathematical statements. Meanwhile, Griffiths [4] states that mathematical proof is a formal and logical way of thinking that starts with axioms and moves forward through logical steps to a conclusion. In addition, proof is also a major component of understanding mathematics [5]. Proof is recognized as the core of mathematical thinking [6]. One cannot study mathematics without studying mathematical proofs and how to make them [7].

The role of proof for a mathematics learner as a determinant of the level of maturity in the process of thinking mathematics [8]. This is because proof requires a person to use mathematical knowledge and write it down in a logical argument, so it requires a comprehensive mathematical thinking process [9]. Recently, several universities have begun to introduce lectures on the introduction of proof or mathematical reasoning programs [10][11], which aim to make it easier for students to understand the formal language of mathematics and its axiomatic structure. This can be seen in the first year students at Universitas Negeri Surabaya where this research took place, because the majority of students have been provided with the initial lecture program, namely in the fundamentals of Mathematics and number theory lectures. Clark & Lovric [12] say that in the process of transitioning into constructing mathematical proofs for students there are many challenges to be faced. They suggest that this transition requires students to change the type of reasoning used, namely shifting from informal to formal language; for reasons of using mathematical definitions; to understand and apply theorems; and make connections between math objects.

Various research results have concluded that the learning process regarding proof of university students has not reached the optimal stage as expected [13-16].The research results of Reiss and Renkl [17] revealed that there were still many student limitations in the proving process. Furthermore, Maarif, Perbowo, Noto, and Harisman [18] concluded from the results of their research that the limitations of student concepts in constructing geometric proofs included difficulties in sketching diagrams with proper geometric labels and difficulties in constructing conjectures in writing formal proofs. In addition, Moore [19] also said that students were unable to understand and use language and mathematical notation in compiling proof. From this, it is necessary for us to optimize the process of exploring the ability to construct proof in order to improve preservice teachers' level of proof in geometry.

Proof in mathematics consists of several universally accepted methods. The methods used in the proof are divided into 2, namely the deduction method and the induction method [5][20]. Proof is recognized as the core of mathematical thinking and deductive reasoning [1]. In deductive proof, a conclusion must be true if the premises are true [21]. The deduction method involves several methods such as direct proof, proof with contraposition and proof with contradiction [22]. Whereas in inductive proof, arguments whose conclusions are not necessarily true but are very likely to be valid [21]. Miyazaki [23] classifies proof into four levels, namely Proof A, Proof B, Proof C, and Proof D. According to Miyazaki [23], Proof A is a level of proof that involves deductive reasoning and functional language used in working on the proof, Proof B is a level of proof that involves deductive reasoning and does not use functional language, images, or manipulation of objects that can be used in the process of doing a proof. Whereas Proof C is a level of proof that involves inductive reasoning and does not use functional language, images, or manipulation of objects that can be used in the process of making proofs, Proof D is a level of proof that involves inductive reasoning and functional language used in doing proofs.

Miyazaki's [23] research explains more about levels in algebra, but in this study the focus will be on geometry. Furthermore, according to Rahayu & Cintamulya [24], teachers need scaffolding through Hypothetical Learning Trajectory (HLT) as a strategy to help student difficulties in proving group problems so that students can increase their level from informal to formal. Anghileri [25] divides the scaffolding hierarchy into three levels in learning mathematics. In scaffolding Level 1 is the most basic level. At this level, a suitable learning environment is needed that can support the learning process. Level 2 in scaffolding is known for several types, namely explaining, reviewing, and restructuring. Assistance provided at that level is used by students to achieve understanding. Level 3 in scaffolding is conceptual development, namely the level of scaffolding that develops concepts students already understand to build connections between concepts.

Based on the description above, this study aims to investigate pre-service mathematics teachers' proof level and possible task of scaffolding-based interventions in proving the triangle theory.

2. RESEARCH METHOD

The method used in this research is a preliminary study of design research. Researchers followed three research phases [26], namely the initial design stage (preliminary design), design testing through preliminary teaching and teaching experiments, and the retrospective analysis stage. In this article, the focus of the discussion is only in the initial design stage (preliminary design). To explain a preliminary study, the researcher uses descriptive research using quantitative and qualitative approaches. At the preliminary stage, the researcher wanted to look at preserve teachers' levels of understanding of proof and preserve teachers' learning trajectories. The participants involved in this study were 58 second-semester mathematics education students at Universitas Negeri Surabaya, Indonesia. There were 2 classes, in which each class consisted of 29 prospective teachers. The choice of research location was based on the curriculum structure of the research location. There is a Basic Geometry course that accommodates proving geometry as an outcome of the learning process. In addition, the selection of the research location was carried out at the author's institution on the grounds that from previous experience teaching geometry, there were still many students who had difficulty constructing of proof.

The data collection technique to see teachers' levels of understanding of proof was carried out by giving a mathematical proof test to 58 students. The data was taken from the results of student work after the lecture process ended, then they were given a 15-minute mathematical proof test to construct geometric proofs. Afterwards, each prospective teacher's responses were assessed to pre-service mathematics teachers' proof level of their deductive and inductive knowledge in constructing a proof.

The present study tends to examine more on deductive and inductive proof without employing interviews like what Miyazaki [23] did. The data were collected using a simple task of constructing one mathematical proof, namely to prove that the sum of the angles in a triangle is 1800. Actually, the task type

Commented [L3]: in the introductory paragraph 5:

Deductive and inductive proof methods are direct proofs. Maybe it can also be explored regarding indirect proof to compare that there are several different methods of proof.

Commented [L4]: The research gap does not appear to be visible in this article. Please disclose the research gaps that have been carried out. This research investigates pre-service mathematics teachers' proof level. Of course, this research has already been done by others. Thus, it is necessary to strengthen what is an important part of the research that has been carried out to complement previous research

Commented [L5]: These 3 phases should be described as this reflects the research being conducted. Each phase must describe how the steps or activities are carried out. Especially in the teaching experiment phase. What kind of treatment was given to respondents in the research that had been done.

Commented [L6]: Methodology section on phalagraph 1 : Pre service teachers or prospective teachers?

Commented [L7]: Methodology section on phalagraph 1 : How are geometry courses done? how long does geometry courses take?

Commented [L8]: Methodology section on phalagraph 2 : The data is visible only from the results of the verification test, even though there are 3 phases in the research methodology. This research seems to only show how the obstacles to the ability to prove by not paying attention to other parts. Though there are 3 phases that have been defined. This is important because it will be incorporated in the results and discussion section could be more than one, such as the sum of the three external angles of a triangle is 3600, or prove the sum of the measures of the angles -the angle of a pentagon is 5400. However, the main point of this study was a proof method whether using deductive proof or inductive proof at each level of proof.

Commented [L9]: Methodology section on pharagraph 3: Fix the writing of the "angle degree" symbol

The process of assessing student answers is carried out by providing scoring coding following the level of proof of Miyazaki's classification [23] in constructing a geometric proof. Because this study using the subject of early mathematics education students who had received both methods in high school [27] and these two methods have often been used by previous researchers in constructing a proof at the university level [28]. Furthermore, researchers try to make a students learning trajectory (LT) for constructing a proof. This LT has not yet been tested on small-scale subjects, it was only made based on learning possibilities that can be used in constructing of proof.

Table 1. Levels of proof in mathematics (Miyazaki, 2000)

Representation	Method	
	Deductive	Inductive
Using functional language according to the theorem	Proof A	Proof D
Do not use functional languages, use images, or manipulate objects	Proof B	Proof C

Proof A was the level of proof when deductive reasoning was involved and a functional language was used in the course of making a proof. Proof was the level of proof where deductive reasoning was involved and other languages, drawings, and movable objects were used in the course of making a proof. Proof C was the level of proof where inductive reasoning was involved and other languages, drawings, and movable objects were used. Proof D was the level of proof where inductive reasoning was involved and a functional language was used.

In the preparation phase of the experiment activities (preparing for the experiment), the researcher designs the Hypothetical Learning Trajectory (HLT) for the learning of proof geometry material. HLT contains learning objectives (mathematical goals), teaching and learning activities, and the conjecture of student thinking. The purpose of this stage is to prepare research including theoretical preparation, designing HLT, making HLT supporting instruments, site preparation, and research subjects. In the expert review activity, the instrument was reviewed by 2 experts who were lecturers from various universities with relevant knowledge. The selection of experts considers the length of service as a lecturer, the level of education, and the quantity and quality of research that has been carried out.

3. RESULTS AND DISCUSSION

Based on triangulation technique that is data collecting technique through interview, observation, and document data, the findings of the research can be seen according to the focus decided already at the beginning of the research, namely:

Commented [L10]: The results section should follow the established method with three phases, namely the initial design stage (preliminary design), design testing through preliminary teaching and teaching experiments, and the retrospective analysis stage. Each phase should describe the results. Thus, the research will be more comprehensive

1.1. Preservice teachers' levels of understanding of proof

In this study, data was collected through a mathematical proof test to see pre-service mathematics teachers' proof level in geometry based on Miyazaki's [23] classification. The results of this mathematical proof test (see Table 2) will be explained as follows:

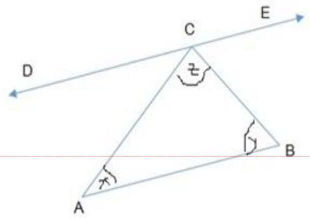
Table 2. Teachers' proof level in geometry (Miyazaki, 2000)

Level	Total of students	Percentage (%)
Proof A	22	38
Proof B	3	5
Proof C	9	15
Proof D	24	42

Based on Table 2, Proof D is the highest score for the level of preserve teachers' answers, namely 24 preserve teachers' answers. This shows that there are still many preserve teachers' answers with non-formal evidence. Furthermore, this Table also depicts that 38% of the preserve teachers performed Proof A in which this proof required deductive reasoning and functional language used to construct proofs. Meanwhile, 5% of the preserved teachers conveyed Proof B with deductive reasoning and manipulating objects or using a sentence without functional language in proof. 15% of the preserve teachers showed Proof C in which they used inductive reasoning and other languages, images, and manipulated objects to construct proofs. Moreover, 42%

of the preserve teachers showed Proof D, in which they used inductive reasoning and functional language for constructing proofs. The following will show some examples of preserving teachers' answers.

Proof A

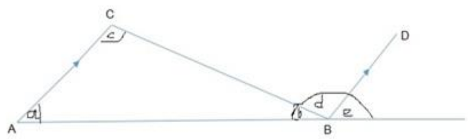


Buat sebuah segitiga sebarang dan beri nama tiap titik sudutnya A, B, dan C. Buat garis yang sejajar sisi AB dan melalui C. dan beri nama garis tersebut. Dalam kasus ini diberi nama DE.

Sudut CAB bersebrangan dengan sudut ACD, sudut CAB = sudut ACD = x^0.
Sudut ABC bersebrangan dengan sudut BCE, sudut ABC = sudut BCE = y^0.
Dan besar sudut ACB yaitu z^0. Sehingga jumlah sudut ACD + ACB + BCE = $x^0 + y^0 + z^0 = 180^0$.

Figure 1. *Proof A*

Proof B



Jadi, karena ketiga sudut itu terletak pada garis lurus maka jumlahnya yaitu 180^0

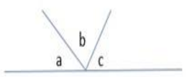
Figure 2. *Proof B*

Proof C

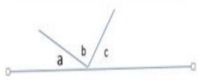
$30^0 + 60^0 + 90^0 = 180^0$
$45^0 + 65^0 + 70^0 = 180^0$
$60^0 + 60^0 + 60^0 = 180^0$
Jadi, jumlah ketiga sudut dalam segitiga sama dengan 180^0

Figure 3. *Proof C*

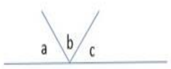
Proof D



($\angle$ a,b,c adalah $\angle$ kurang dari 90^0)



($\angle$ a,c adalah $\angle$ kurang dari 90^0 dan $\angle$ b = 90^0)



($\angle$ a,b,c adalah sama dan $\angle$ kurang dari 90^0)

Figure 4. *Proof D*

Commented [L11]: In the picture proof A, that has been shown in the Proof A section, it is written that the conjecture AB//DE, but in the picture you can't see the symbol that the two are parallel to each other. Please include a symbol stating that AB//DE in order to strengthen the allegation that has been made.

Commented [L12]: In pictire proove A, Please translate the proof questions into English/ there is a translation in English so that readers can find out the problem to be proven

Commented [L13]: In pictire proove A, In the proof section it is written "Sehingga sudut ABC+ACB+BCE = ……" based on the concept that addition cannot be operated on, which can be operated quantitatively or angles have no size. Which has a size is "angle size" so it would be better if the statement is corrected to "Sehingga jumlah besar sudut ACD+ besar sudut ACB+ besar sudut BCE =…..''or can use the right mathematical symbols.

Commented [L14]: In picture proof A, B, C, Please translate the proof questions into English/ there is a translation in English so that readers can find out the problem to be proven

3.2 Preservice teachers' learning trajectories using scaffolding in Geometry

Based on expert comments, researchers arrange things that are considered necessary to be discussed with experts. In outline, two things are subject to discussion between researchers and experts: students' understanding of the use of four levels of proof (Proof A,B, C, and D) that will be used in HLT activities and the need to make separate steps. HLT, which is arranged as an initial design is called the initial prototype. The initial prototype HLT consisted of four teaching-learning activities: reading proof, completing proof, evaluating proof, and constructing proof. In the expert review activity, the researcher intends to obtain an expert judgment on the relevance of the activities to achieve the expected goals along with the researcher's hypothesis about the conjecture of students' thinking. After the discussion with the experts, the following revision materials for the initial prototype HLT are in Table 3.

Table 3. The HLT of proof with Scaffolding in Geometry

No	Activity	Goals	Students conjuctured thinking	Type of scaffolding
1	Reading Proof	The purpose of the first activity "Reading Proof" is to introduce the parts that must be present in the sentence of proof and the levels of proof in constructing proof (deductive reasoning).	1. Read carefully the proof of the following basic geometry theorems (Students are given complete proof, inductive proof for answer a question with Proof C) 2. After reading the proof of the theorem, then write down the premises (statement / closed sentence) of each statement of proof! 3. After reading the proof of the theorem, then write down the things you have understood (in a few points, if any) in the box below! 4. From the results of the class, discussion write in full the conclusions / new understanding that you get (If any)!	With student difficulties do not know how to start the proof Level 2 (*explaining, reviewing, dan restructuring*)
2	Completing Proof	The purpose of the second activity "Completing Proof" is to train students to identify sentences/statements of proof that must be present in the proof sentence (incomplete), the use levels of proof in constructing the proof.	1. Read carefully the proof of the following basic geometry theorems! (Students are given incomplete proof, inductive proof for answer a question with Proof D). 2. After reading the proof of the theorem in point 1, then write down the things that you think are incomplete (if any) of the proof of the theorem! 3. Write the complete proof of the theorem on point 1!	With limitations of students do not understand and use language and mathematical notation Level 2 (*explaining, reviewing, dan restructuring*.)
3	Examining Proof	The purpose of the third activity "Evaluating Proof" is to train students to evaluate the sentences/statements of proof presented by identifying errors	1. Read carefully the proof of the following basic geometry theorems! (Students are given proof by logic/ wrong correct concept, deductive proof for answer a question with Proof B) 2. After reading the proof of the theorem, then write the things that are FALSE in your opinion (if any) in the box below! 3. Write the right proof of the theorem on point 1!	With difficulty understanding the concept students Level 2 (*explaining, reviewing, dan restructuring*)
4	Constructing proof	The purpose of the fourth activity "Constructing Proof" is to train students to construct their sentences/statements of proof from several theorems provided with	1. Read carefully the proof of the following basic geometry theorems! (Students are given proof by logic/wrong concept, deductive proof for answer a question with Proof A) 2. In your opinion, the theorem in point 1 is more effectively proven using deductive	With difficulty understanding the concept students Level 2 (*explaining, reviewing, dan restructuring*) or Level 3 (*conceptual development)*

	the correct sentence and proof of logic	proof or inductive proof? Explain your reasons! 3. Write the right proof of the theorem on point 1!

Preservice teachers learning trajectories are shown in Table 3. All preserve teachers' start at level C (see Table 3), because preservice teachers already know some geometric terms, definitions and axioms about angles of triangles from the previous meeting. All preserve teachers' will generally develop in the same way through the main pathways of learning trajectories; Individual differences will also be seen in development time and the degree to which students can be involved in constructing of proof.

The preservice teachers perform Proof A, Proof B, Proof D, and Proof C types with the percentage of 38%%, 5%, 42%, and 15%, respectively. Therefore, it shows that Proof D is the most commonly found in the prospective teachers' answers than those of other types. It aligns with the results of Kögce et al. [5], in which the study results report that the inductive method is performed by most students than the other types of proof (51.2%). The fact that our study has found many inductive methods in our participants' answers might indicate difficulties in starting of proof, understanding concepts, and using symbols or language in compiling of proof. In line with Baker's research [29], many students experience difficulties in using symbols in constructing a proof. Harel & Sowder [30] also concluded that many students had difficulty coming up with invalid deductive arguments and inductive arguments. Based on these difficulties, a learning trajectory is needed in the form of scaffolding to assist students in compiling a proof.

The present study not only indicates the level of prospective teachers regarding the classification of Miyazaki [23] in proving processes, but also shows the student learning trajectory in constructing proof. This student learning trajectory was developed in order to get a broader insight on how to evaluate the level of proof and proving difficulties from a written response representing an individual proof task. It is expected that this analytical students learning trajectory will complement other learning in mathematics so that students can understand a concept or solve a mathematical problem.

The results of other studies that support the division of levels of thought and activity to construct proof are research at the level of students' ability to construct proof based on information processing theory [31]. Based on these results it can be concluded that there are four levels of student proof according to the level of students' ability to construct student proof based on information processing theory: Proof C, Proof D, Proof B, and Proof A. The low students are still struggling with processing information and knowledge, and their understanding of the concepts needed in Constructing the proof is still very limited (Proof C). The recommendation for students in this group, according to HLT is the activity of reading proof where students are asked to examine the concepts used in constructing the proof and the logic of the proof. Next, students have succeeded in processing information but failed in Constructing the arguments presented by middle students in the construction of the proof which is unclear and incomprehensible (Proof D). Based on the developed HLT, the recommendation for this students is the activity of completing proof and examining proof where students are trained to apply and analyze the concepts needed in constructing proof. For last students where the information processing component is functioning properly, the activity of Constructing Proof can be recommended (Proof B and Proof A).

4. CONCLUSION

In conclusion, there were 38% of students' answers in constructing of proof with level Proof A, 5% of students' answers in constructing of proof with level Proof B, 15% of students' answers in constructing of proof with level Proof C, and the remaining 42% of students' answers in constructing of proof with level Proof D. Furthermore, this research resulted in a Hypothetical Learning Trajectory (HLT) of Proof Geometry Material which contains 4 activities: reading proof, completing proof, examining proof, and constructing the proof. The scaffolding-based intervention task refers to the preservice teacher's difficulties in proving the triangle theorem, including a lack of understanding of concepts, not understanding language and mathematical notation and difficulties in starting proofs

REFERENCES

[1] Cheng, Y. H., & Lin, F. L. (2009). Developing learning strategies for enhancing below average students' ability in constructing multi-steps geometry proof. In *Proceedings of the ICMI study 19 conference: Proof and proving in mathematics education* (Vol. 1, pp. 124-129). Taipei: National Taiwan Normal University.
[2] Hernadi, J. (2008). *Metoda pembuktian dalam matematika*. Jurnal Pendidikan Matematika, 2(1), 1-14.
[3] Mingus, T. T., & Grassl, R. M. (1999). Preservice teacher beliefs about proofs. *School Science and Mathematics*, 99(8), 438-444.
[4] Griffiths, P. A. (2000). Mathematics at the turn of the millennium. *The American Mathematical Monthly*, 107(1), 1-14
[5] Köğce, D., Aydın, M., & Yıldız, C. (2010). The views of high school students about proof and their levels of proof (The case of

Commented [L15]: The results and discussions section should follow the established method with three phases, namely the initial design stage (preliminary design), design testing through preliminary teaching and teaching experiments, and the retrospective analysis stage. Each phase should describe the results. Thus, the research will be more comprehensive

Commented [L16]: The results reveal "The preservice teachers perform Proof A, Proof B, Proof D, and Proof C types with the percentage of 38%%, 5%, 42%, and 15%, respectively. ". Where did this result come from? whether from errors or the number of respondents who answered correctly? does it refer to errors based on HLT table 3?. If possible, data can be displayed in tabular form and a representation of that percentage

Commented [L17]: The conclusion that says "The scaffolding-based intervention task refers to the preservice teacher's difficulties in proving the triangle theorem, including a lack of understanding of concepts, not understanding language and mathematical notation and difficulties in starting proofs ". How this conclusion can be constructed. Is there a conformation process with interviews or just observation. This is important to reveal and discuss in the results and discussion sectio

Commented [L18]: Akan lebih baik, jika di tampilkan rekomendasi dan limitasi dari penelitian yang telah dilakukan

Trabzon). *Procedia-Social and Behavioral Sciences*, 2(2), 2544-2549.
[6] Hanna, G., De villiers, M., Arzarello, F., Dreyfus, T., Durand- Guerrier, V., Jahnke, N.H., (...) Yevdokimov, O. (2009). ICMI Study 19: Proof and Proving in Mathematics Education (Discussion Document). In Lin, Fou-Lai (Eds.). *Proceeding of The ICMI Study 19 Conference: Proof and Proving in Mathematics Education* (vol.1). Taipei: The Department of Mathematics, National Taiwan Normal University
[7] Balacheff, N. (2010). Bridging knowing and proving in mathematics: A didactical perspective. In *Explanation and proof in mathematics* (pp. 115-135). Springer, Boston, MA
[8] Otten, S., Gilbertson, N. J., Males, L. M., & Clark, D. L. (2014). The mathematical nature of reasoning-and-proving opportunities in geometry textbooks. *Mathematical Thinking and Learning*, 16(1), 51–79
[9] Cervantes-Barraza, J. A., Hernandez Moreno, A., & Rumsey, C. (2020). Promoting mathematical proof from collective argumentation in primary school. *School Science and Mathematics*, 120(1), 4–14.
[10] Epp, S. S. (2003). The role of logic in teaching proof. *The American Mathematical Monthly*, 110(10), 886-899.
[11] Selden, A., & Selden, J. (2007). *Overcoming students' difficulties in learning to understand and construct proofs* (Report No. 2007-1). Cookeville: Mathematics Department, Tennesse Technological University
[12] Clark, M., & Lovric, M. (2008). Suggestion for a theoretical model for secondary-tertiary transition in mathematics. *Mathematics education research journal*, 20(2), 25-37
[13] Jones, K. (2010). Proving in geometry. *Proceedings of the British Society for Research into Learning Mathematics*, 30(2), 62-67.
[14] Michael, P., Gagatsis, C., & Gagatsis, A. (2013). Geometrical figures in geometrical task solving: an obstacle or a heuristic tool?. *Acta Didactica Universitatis Comenianae*. Mathematics, 2013(13), 17–32.
[15] Daguplo, M. S., & Development, H. R. (2014). How well do you write proof? characterizing students proof - writing skill vis - à - vis van hiele's model in geometrical proving. *Journal of Educational and Human Resource Development*, 2, 104-114
[16] Azrou, N., & Khelladi, A. (2019). Why do students write poor proof texts? A case study on undergraduates' proof writing. *Educational Studies in Mathematics*, 102(2), 257-274
[17] Reiss, K., & Renkl, A. (2002). Learning to prove: The idea of heuristic examples. *ZDM - International Journal on Mathematics Education*, 34(1), 29–35
[18] Maarif, S., Wahyudin, Noto, M. S., Hidayat, W., & Mulyono, H. (2018). Geometry exploration activities assisted with Dynamic Geometry Software (DGS) in a teacher education classroom. *Infinity Journal*, 7(2), 133-146.
[19] Moore, R. C. (1994). Making the transition to formal proof. *Educational Studies in Mathematics*, 27(3), 249-266.
[20] Siswono, T. Y. E., Hartono, S., & Kohar, A. W. (2020). Deductive or Inductive? Prospective Teachers' Preference of Proof Method on an Intermediate Proof Task. *Journal on Mathematics Education*, 11(3), 417-438.
[21] Anderson, J. R. (1985). *Cognitive psychology and its implications (2nd ed.).* New York: WH freeman
[22] Morali, s., Uğurel, i., Türnüklü, e., & Yeşildere, s. (2006). Matematik öğretmen adaylarinin ispat yapmaya yönelik görüşleri. *Kastamonu Eğitim Dergisi*, 14(1), 147-160.
[23] Miyazaki, M. (2000). Levels of proof in lower secondary school mathematics. *Educational Studies in Mathematics*, 41(1), 47-68.
[24] Rahayu, P., & Cintamulya, I. (2021, February). Scaffolding as a strategy to help student difficulties in proving group problems. *In Journal of Physics: Conference Series* (Vol. 1776, No. 1, p. 012037). IOP Publishing.
[25] Anghileri, J. (2006). Scaffolding practices that enhance mathematics learning. *Journal of Mathematics Teacher Education*, 9(1), 33-52
[26] Gravemeijer, K., & Cobb, P. (2006). Design research from a learning design perspective. In *Educational design research* (pp. 29-63). Routledge.
{27] Brady, P., & Bowd, A. (2005). Mathematics anxiety, prior experience and confidence to teach mathematics among pre-service education students. *Teachers and teaching*, 11(1), 37-46.
[28] Almeida, D. (2001). Pupils' proof potential. *International Journal of Mathematical Education in Science and Technology*, 32(1), 53-60.
[29] Baker, J. D. (1996). Students' difficulties with proof by mathematical reasoning. In *Annual Meeting of the American Educational* Research Association, New York.
[30] Harel, G., & Sowder, L. (1998). Students' proof schemes: Results from exploratory studies. *American Mathematical Society*, 7, 234-283.
[31] Agustiani, R., & Nursalim, R. (2020). Hypothetical learning trajectory (HLT) for proof logic topics on algebra course: What're the experts think about?. *Al-Jabar: Jurnal Pendidikan Matematika*, 11(1), 101-110.