Introducing geometry concept based on history of Islamic geometry

by Samsul Maarif

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Introducing geometry concept based on history of Islamic geometry

S Maarif[1,2], Wahyudin[2], A Raditya[3] and K S Perbowo[4]

[1]Department of Mathematics Education, Universitas Muhammadiyah Prof. DR. HAMKA, Indonesia
[2]Department of Mathematics Education, Universitas Pendidikan Indonesia, Indonesia.
[3]Department of Mathematics Education, Universitas Muhammadiyah Tangerang, Indonesia
[4]Department of Mathematics Education, Universitas Muhammadiyah Prof. DR. HAMKA, Indonesia

Email: samsul_maarif@uhamka.ac.id

Abstract. Geometry is one of the areas of mathematics interesting to discuss. Geometry also has a long history in mathematical developments. Therefore, it is important integrated historical development of geometry in the classroom to increase' knowledge of how mathematicians earlier finding and constructing a geometric concept. Introduction geometrical concept can be started by introducing the Muslim mathematician who invented these concepts so that students can understand in detail how a concept of geometry can be found. However, the history of mathematics development, especially history of Islamic geometry today is less popular in the world of education in Indonesia. There are several concepts discovered by Muslim mathematicians that should be appreciated by the students in learning geometry. Great ideas of mathematicians Muslim can be used as study materials to supplement religious character values taught by Muslim mathematicians. Additionally, by integrating the history of geometry in teaching geometry are expected to improve motivation and geometrical understanding concept.

1. Introduction

Since geometry has an important role in mathematics and sciences history, so the process of teaching and learning geometry should be done innovatively. Innovative learning in geometry can also develop a positive attitude of students towards geometry. Therefore, the teaching of geometry should be done in an environment that motivated students to discover knowledge independently. It is the reason that makes the development of geometry should be accompanied by activities to improve the quality of learning. Revealed that learning mathematics should be continuously developed accordance the technology development so that it can improve student learning outcomes [1].

Revealed that learning mathematics (including geometry) effectively can be done by integrating the history of mathematics into mathematics lesson [2]. History of geometry may be as old as a human civilization. A brief history of geometry is important to explore the development of geometry and forming geometry curriculum in schools, it is important that we know a brief history of geometry from time to time. Teacher requires knowledge of the history of geometrical concept so that the student can learn not only the geometry knowledge but also the philosophical values of geometry concepts.

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Liu presented five reasons why the history of mathematics should be included in mathematics lessons. These reasons are: 1) historical knowledge increases students' motivation, 2) helps them to develop a positive attitude towards mathematics, 3) seeing the obstacles experienced in the development of mathematics in the past allows them to see difficulties encountered at present, 4) to solve problems from history helps the development of students' mathematical thinking, 5) history brings out the human side of mathematical knowledge and it is a guide to teachers [2]. On the other hand, Katz reveals that usage wisely history of mathematics (including geometry) can be a medium of learning and can assist teachers in creating more effective learning. In particular, the integration of the history of mathematics (including geometry) in learning not only can improve students' understanding but also can stimulate communication, connection skill and improve students' appreciation in mathematics [3].

Jones revealed that teaching geometry effectively involves, amongst other things, appreciating the history and cultural context of geometry, knowing how to recognize interesting geometrical problems and theorems, understanding the many and varied uses to which geometry is put, and incorporating all these things into the practice of teaching in the classroom [4]. Zebrowski revealed that by teaching what has been learned in the past, allow students to apply the past knowledge on the knowledge in the future. We cannot teach or learn if we did not believe that the universe represents the continuity of history [5].

For teachers, history of geometry is an important part to understand geometry. If teachers know the origins and context of a geometrical concept, then it will be easier for them to make a learning material more related to their everyday life. According to Gursoy, pre-service teachers have developed a positive attitude towards the history of mathematical teaching and learning process. There are many things that can be learned from the history of mathematics to prepare a lesson plan that will be implemented in the classroom [2].

The importance of teaching geometry with attention to the history of the development, also with regard to the students' thought process. In learning, geometry either as a product or a process of thinking. According to Castagnola, there are two ways of thinking about mathematical knowledge: either as a product or a process. Thinking about mathematical as a product means being concerned with the results and the structure of that knowledge, that is to say, with mathematical discourse [6]. Thinking about mathematical as process means being concerned with mathematical activity. A history of mathematics centered on problems brings to the fore the process of the construction and rectification of knowledge arising out of the activity of problem-solving.

In Indonesia, history of Islamic geometry is less popular in the mathematics curriculum. One of the reasons for this situation because the reference about Islamic geometry is not widely distributed. Primary to high school textbook rarely place the Muslim mathematicians work. In the college level, study of the history of Islamic geometry are less studied. In fact, the history of Islam in particular geometry and the history of Islamic mathematics, in General, is not included in the existing curriculum in Indonesia.

The objectives of educational activity that contained in Constitution No.20/ 2003, revealed that the purpose of education is to bring about a human being who has the power of religious, spiritual self-control, personality, intelligence, morals, as well as the necessary skills themselves, the community, the nation and the State. Then, the geometrical teaching and learning by integrating historical development of Islamic geometry are very precise. Students not only learn geometry but also have a positive attitude of Muslim mathematicians who have contributed to the development of geometry.

In the development of geometry, there are some geometric concepts invented by Muslim mathematicians. As has been expressed by Maarif that by an analogy to the case in the present study, so students can be trained to see the extent to which the concept can understand, see the microscopic structure of a concept by researching the connections between the cases presented with concepts that are learned and open the minds of students about the applications and benefits of the concepts he had learned [7].

The idea of a Muslim religious studies material can be used to add the values of the characters. By integrating the history of geometry in a geometry learning is expected to add to the learning motivation of students to understand the concepts of geometry. As has been expressed by Maarif, the geometry is a branch of mathematics that studies the form of objects and their characteristics [8]. Geometry is a representation of the universe in mathematics. Surely the universe contains many meanings that must be revealed by geometric approach in mathematics. The existence of the people of the world on the basis of the existence of a God with nature *rakhman* and *rakhim*.

From above explanation, it's important to understand geometric concept and history from the perspective of Islamic mathematician. Hope this new perspective will improve the quality of teaching and learning geometry.

2. Methods

The research using literature research methodology. Literature research methodology is to read through, analyze and sort literatures in order to identify the essential attribute of materials [9]. In this research, literature material is containt a various research finding about history and concept of geometry that developed by Islamic mathematician.

3. Results and Discussion

3.1. History of Geometry Periods

Jones revealed that history of geometry indicates the following three distinct periods: Intuitive, Classical, and Modern geometry [10].

3.1.1. Intuitive Geometry (2000 B.C - 500 B.C)

The periodical inundation of the Nile which swept away the landmarks in the valley of the river, and by altering its course increased or decreased the taxable value of the adjoining lands, rendered a tolerably accurate system of surveying indispensable, and thus led to a systematic study of the subject by the priests [11]. Smith writes that the earliest geometry "was intuitive in its nature; that is, it sought facts relating to mensuration without attempting to demonstrate these facts by any process of deductive reasoning" (p.270) [10]. In earliest geometry nature was the main problem to solve intuitively. That experiences not only transferred but also gradually develop from one generation to another. Around 1700 BC, the very important manuscript called "Ahames" papyrus was written. This manuscript consists of many techniques to solve arithmetic problem and mensuration, areas of rectangles and circles [12].

3.1.2. Classical Geometry (500 B.C -1600 A.D)

In this period, Greek mathematicians more dominant than Babylonian and Egyptian mathematicians. Because of that reason the geometry was not focused to solve a practical problem but as a mathematical concept. The Greek mathematician using geometry as a concept to understand the universe. The Greek mathematicians such as: Thales, Pythagoras, Eudoxus and Euclid produced a masterpiece in geometry. Yazdani writes that in this period the geometry idea was demonstrated and formalized systematically as a mathematical concept [10]. Thales in early sixth century was the first Greek mathematician that using a deductive reasoning to prove a geometrical problem. The contribution of Thales is not lies in the discovery of theorem, but in their proof. In 300 B.C, Euclid writes the "Element". It is a collection of definitions, postulates (axioms), propositions (theorems and constructions), and mathematical proofs of the propositions. After over 2000 years this book is still the most influential textbook ever written [13].

While Europe endured its "Dark Ages" in 6th to 14th centuries, the Middle East became Muslim and in high point of Islamic civilization. In early 9th century under Caliph Harun al-Rashid and (continued by his son) al-Ma'mun, The House of Wisdom (bayt al-hikma) was established and became a center for the study of science and humanities. The scholar translating a large number of Greek, Hindu, Syriac-Persian, and Hebrew texts into Arabic. Al-Khwarizmi, Abu Wafa, and Omar Khayyam were great names that joined and worked in the House of Wisdom.

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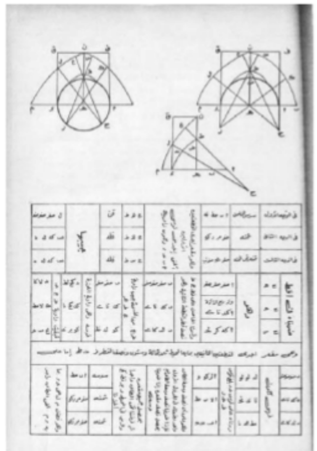

Figure 1. Page from *Al-Kashi* [15]

3.1.3. Modern Geometry (1750 M- now)

Eves revealed that the modern geometry period began when the attempted to prove Euclid's fifth postulate. Euclid established five axioms and five postulates for geometry but in the fifth postulate, better known as Euclid's parallel postulate, has become one of the most controversial statement in mathematical history. For roughly two millennia there were attempts to prove the postulate [8]. Recorded efforts were made by Proclus (fifth century), Thabit ibn Qurra (ninth century), ibn al-Haytham (tenth century), Khayyam (eleventh century), Nas.ir al-Din al-Tusi (thirteenth century); and, in 'modern' times like Playfair, Legendre, Gauss, Bolyai, Lobachevsky, Beltrami, Klein, Poncelet, Poincare, and Riemann. When many mathematicians tried to prove the Euclid's parallel postulate, few of them (such as: Saccheri, Beltrami and Lambert) using alternative postulate developed non-Euclid geometry [14,10].

3.2. Muslim mathematician and Geometry

3.2.1. Mohammad Abu'l-Wafa

Mohammad Abu'l-Wafa al'Buzjani (940 – 988) called al-Wafa born at Buzjan in Khorasan region where Iran is today. He was one of the most productive and well-known Islamic mathematicians and astronomers. He did most of his work at Baghdad observatory or the "House of Wisdom". His known for his translation and commentators on Greek works such as Diophantus, Euclid and al-Khwarizmi. He also was developed concept of trigonometry, arithmetic, study orbit of the moon and introduced the sec and cosec and studied the interrelations between the six trigonometric lines associated with an arc [15].

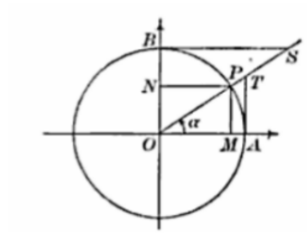

Figure 2. Page from *Al-Kashi*

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PM= Sin; TA= tangent; OT = Secant; NP= Cos; RS = Cotangent and OS= Cosecant

Al-Wafa's contributions to trigonometry also proving of trigonometry sum-difference identities [16]:

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

And from which we can get the double angle identities:

$$\sin(2\alpha) = 2\sin\alpha\cos\alpha$$
$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$$

Another contribution from al-Wafa to geometry was found in Kitab Al-Hindusa. This book consisted of solutions of a large number of geometrical problems on the fundamental construction of plane geometry with the opening of the compass [17]. Al-Wafa used his dissection and construction method to create a novel geometric proof of the Pythagorean theorem. Two unequal squares are added together to make a third square [18].

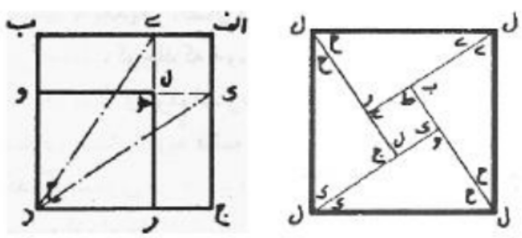

Figure 3. Drawing from original Persian text

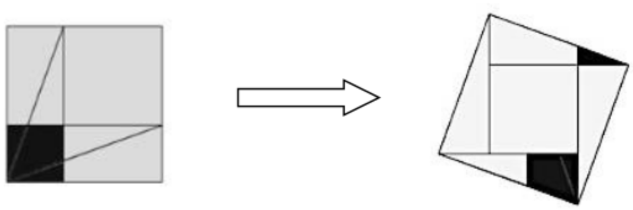

Figure 4. Geometric proof of the Pythagorean theorem

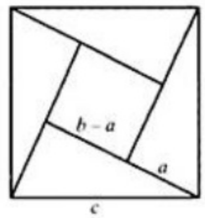

Figure 5. Geometric proof of the Pythagorean theorem

In al-Wafas' proof, a small square is placed to share the corner and two side segments of a larger square. Then a larger triangle is built up by dissecting both the large square and the small square and adding the large square to the small square to make a bigger square. In Figure 4, the area of small black square is equal to a^2, the area of large grey square superimposed behind the small black square is equal to b^2, and the area of larger square made from adding a^2 and b^2 is equal to c^2 (See Figure 4). In the final square, as shown in Figure 5, the edge a, b, and c can be viewed as the hypotenuse and other sides of a right triangle taking up part of the square [18].

3.2.2. Al-Khwarizmi

Al-Khwarizmi from Baghdad (ca. 780 – 850) is probably the most famous Muslim mathematician. His contribution not only in mathematics conc such as (number of mathematical concepts, geometry, arithmetic, algebra) but also in astronomy. In his book Al-kitāb Al-Mukhtaṣar fī Hisāb Al-Gabr Wa'l-muqabala, Al-Khwarizmi created the method of algebra equation hand in hand with the area of the square. Al-Khwarizmi also demonstrated a geometry approach to solve the algebra problem, this method now we call "completing the square" [19]. If he had an equation:

Figure 6. Page from Al-kitāb Al-Mukhtaṣar fī Hisāb Al-Gabr Wa'l-muqabala

$$x^2 + 10x = 39$$

He would create a small square on each side has a length that equal to x.

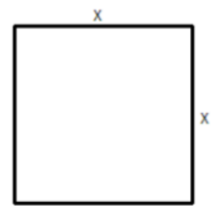

Figure 7. Square with x length

This square has an area equal to

$$x^2 = 39 - 10x$$

Then he added a rectangle to each side of the square, with a width of $\frac{10}{4} = 2.5$ unit.

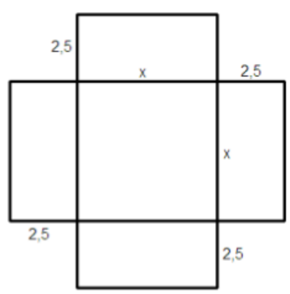

Figure 8. Modified Rectangle

So, the area of a rectangle's in Figure 8 has an area equal to 39 units. In other word:

$$x^2 + 4\,(2.5x) = x^2 + 10x = 39$$

Next, he added a line in corner between both rectangles, so each corner became a square with side length equal to 2.5 units.

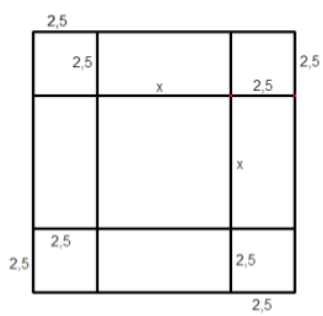

Figure 9. Modified Rectangle

The rectangles has a length equal to $x + (2{,}5 + 2{,}5) = x + 5$ unit. Hence, the area of the rectangle was $(x + 5)^2$ units. In the other hand, the area of rectangle in picture 8 is 39 units and the square in corner (picture 9) has an area equal to 6.25 units each. Hence the total area for rectangle in picture 9 equal to 39 + 4 (6.25) = 64 units. From both statement we can assume that $(x + 5)^2 = 64$ units, using simple arithmetics the x value equal to 3 units.

3.2. Abu Nasr al-Farabi

The Center for South Asian and Middle Eastern Studies, University of Illinois at Urbana-Champaign revealed that Islamic geometry also produced some concept in geometry. Abu Nasr al-Farabi (870 – 950 CE) was one of the Islamic Mathematician that concern about geometry, philosophy and astronomy. He wrote a treatise called "A Book of Spiritual Crafts and Natural Secrets in the Details of Geometrical Figures", in which he talked about several concepts of geometry and how to construct it [20].

Few problems that solved by Aby Nasr al-Farabi were 1) To construct at the endpoint A of a segment AB a perpendicular to that segment, without prolonging the segment beyond A and 2) To divide a line segment into any number of equal parts (for example, three equal parts).

Problem 1: "To construct at the endpoint A of a segment AB a perpendicular to that segment, without prolonging the segment beyond A".

1) Create segment AB and put point C anywhere in segment AB
2) Construct a circle in A with radius AC and construct a circle in C with radius AC. Hence an intersection from both circle in D.
3) Construct a circle in D with radius BD. Create a line CD, intersection between line and circle is E.
4) Hence AE is perpendicular with AB in A.

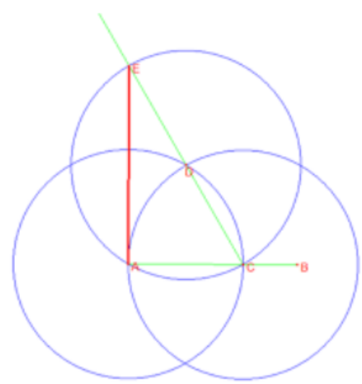

Figure 10. Problem 1 Abu Nasr al-Farabi

Problem 2: Divide a line segment into any number of equal parts (for example, three equal parts).

1) Create segment AB and put point C as a midpoint segment AB
2) Construct a circle in A with radius AC
3) Construct a circle in B with radius BC
4) At both endpoint creates perpendicular AD = BE
5) Add midpoint F and G hence AF=FD=BG=GE
6) Create segment DG and FE. Both segment intersecting AB in H and I.
7) Hence segment AB divided into three equal part in which segment AH, HI and IB.

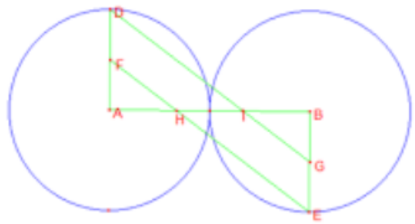

Figure 11. Problem 2 Abu Nasr al-Farabi

3.2.4. Ibrahim Ibn Sinan

Ibrahim ibn Sinan from Asfhana (in 980 CE) was another Islamic mathematician. He was known as a translator of Archimedes. He also constructed a nice and simple step to construct a parabola curve with this following method [21].

1) Create segment AG
2) Put point B between segment AG, on BG put as many points as you want (in this case 5 points: H, D, Z, X, and Y)
3) Create a circle with diameter AH and create a segment BE that perpendicular from segment AG
4) Put point T as an intersection between circle and segment BE.
5) Create line parallel to AG through point T and create line parallel to BE through H, hence the intersection was point K
6) Create a circle with diameter AD
7) Put point L as an intersection between circle and segment BE.
8) Create line parallel to AG through point L and create line parallel to BE through D, hence the intersection was point M
9) Follow the same construction method for point Z, X, and Y
10) Create K', M', …. on the extension of the lines KH, MD and … so that KH = HK', MD = DM' …. Then K', M', …. Also lie in parabola.

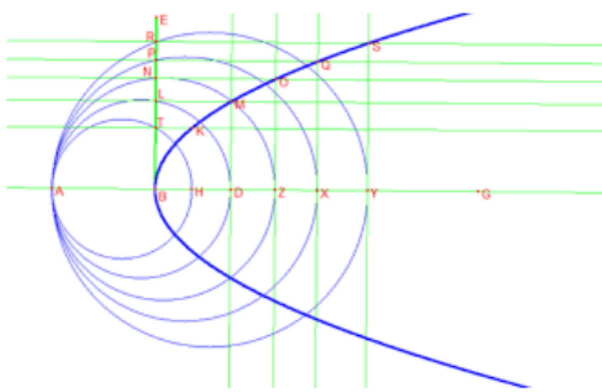

Figure 12. Parabola's construction from Ibn Sinan

4. Conclusions

Geometry lesson with historical perspective not only will provide a deep understanding for student but also can motivate them to learn geometry. The various example and story from historical perspective is good way for students to understand that geometrical problem is also everyday problem. Then students realise that problem can be solved with logic, persistence and an understanding of concept that invented before. Problems from the history of mathematics may be included as different methods in teaching and learning environments. For example, said that history of mathematics could be used in the method of the invention [22].

However, for the purpose of this paper, it suffices that the Muslim mathematicians also had a great part in development of geometry. Indonesian, as a biggest Islamic community in the world, needs to consider embedding history of Muslim mathematician in mathematics (especially geometry) curriculum.

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