

KHOERUL UMAM DAN ARDI

by Ijer Scopus

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CRITICAL THINKING SKILLS: ERROR IDENTIFICATIONS ON STUDENTS' WITH APOS THEORY

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ABSTRACT

Purpose: Identifying students' errors solving cases of critical thinking skills from two variables of linear equations (TVLE).

Research Methods: This was a qualitative study using a descriptive exploratory approach. One university student of the first semester was the participant of the study. The method used in this research is a test, interview, and triangulation.

Findings: The findings showed that the student has low critical thinking skills; therefore, he could not complete the task correctly. Based on the APOS theory, students' mistakes in completing math problems include four elements, namely: (1) the errors in interpreting; (2) the errors in understanding the concept; (3) the error in the procedures; and (4) the error in technical things. The student's response in this study was in the theoretical of APOS so that he could not have a correct schema.

Implications for Research and Practice: The study results are expected to be beneficial in developing the activities in teaching TVLE so that the students will not make any errors in completing critical thinking skills problems in mathematics. Therefore, further study in developing a teaching model for mathematics teaching to improve students' critical thinking skills is highly recommended.

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1. INTRODUCTION

Critical thinking skills become fundamental in the 21st century that educators teachers begin to develop these skills [1], [2]. This is mainly because critical thinking skills are an essential prerequisite for continuing higher education [3], [4]. Moreover, society has considered the critical thinking skills the most augmented intellectual activity occupied in decision making [5]. The study has shown that someone can generate self-motivated knowledge in civilization if one has a critical thinking skill [6]. Having good critical thinking skills can lead one to have cognitive competence to reach the desired objective [7], [8]. Therefore, one of the purposes of the educational system has put critical thinking skill as the one of utmost objective in education [8], [9]. Undoubtedly, everyone needs to develop this skill to make reasonable and reliable decisions to help him cope with life problems [10].

One of the ways to cope with the problems is through critical thinking skills developed in mathematics Participants. This is in line with the statement that developing critical thinking skills has been the primary agenda in the curriculum of mathematics teaching all over the world [2], [11]. This has also been the concern of the Indonesian government to develop critical thinking skills in mathematics, which is an essential competency in the curriculum for students to be successful in the future [1], [12]. However, since joining PISA in 2000 up to 2015, Indonesian students are always in the low rank in terms of their mathematic achievement [13]. In the TIMSS survey, Indonesian students' mathematics achievement is alarming [14]. One contributing factor to the low PISA and TIMSS results is that students are not accustomed to solving mathematics problems that demand critical thinking skills [15]. [16] added that the critical thinking skill in mathematics must be developed to help pupils complete the mathematic cases. Therefore, mathematics and critical thinking must not be separated in creating meaningful learning activities [6], [17].

Critical thinking in mathematics has aroused several expertise to set some indicators for this skill. [18] proposed that one can be categorized as having the critical thinking skill when he has fulfilled six indicators of good critical thinking skill, which cover *focus, reason, inference, situation, clarity*, dan *overview*. *Focus* means able to identify problems of decision-making. Reason means able to provide arguments based on the relevant evidence needed in decision making. Inference means able to correctly infer or conclude based on relevant opinions to support the inference made. The situation means using all the information needed based on a particular circumstance in the problems being faced.

Clarity means able to give a further explanation concerning the conclusion which has been made. Overview means conducting a review of each of the steps in decision making. University students of the mathematics education department have somehow neglected these essential yet crucial indicators. The evidence shows that the critical thinking skill of university students of Indonesia is still not entirely satisfactory [14], [19]. They still have a low degree of support in having critical thinking activities in education [20]. This was caused by several factors, including the students, the instructors, and the learning environment during the teaching and learning process. It was assumed that those who could not manage to have high their critical thinking skill was due to the lack of understanding of the mathematics concepts they have mentally. Therefore they still made many errors in completing the mathematics tasks related to their critical thinking skill. It is advised that the students must have an excellent conceptual understanding of mathematics. [21] proposed that when the students have an incorrect interpretation of their basic mathematics concepts, it will be challenging to develop their ability. The basic concepts in understanding mathematics are to ease them in expanding their knowledge of mathematics itself.

The reality has revealed that many students have difficulties finishing the conceptual understanding [22]. [23] urges that the students were unsuccessful in identifying the causes of misconception, which leads them to have wrong ways of finding practical solutions to ease the misconception itself. The misconceptions that occurred in students have shown them to have misinterpretation, procedural errors, and technical errors. The mistakes in interpretation (reversal) refer to translating or deducing the student's word choice in prescribing variables into numbers. It tends to be in the object rather than numbers [24]. [25] affirmed that the conceptual (structural) errors show the failure to understand a concept within a problem, which also means a failure to value the relationships within a situation [25]. The procedural (arbitrary) errors occurred when a person failed in manipulation or an algorithm, even if he has understood the concept [25], [26]. The errors are related to incorrect choices of extrapolation [27]. For these reasons, it is essential to check the students' mental process in completing the mathematics tasks related to critical thinking skills.

There are many ways to check students' mental processes in critical thinking mathematics, and One is employing the APOS (actions, process, object, and schema) theory. [28] claimed that the APOS theory covers four phases, actions, process, object, and schema. In the action phase in the APOS theory, the transformation is done from the mathematics concept by an individual regarding explicit algorithm and is led by external stimulation [28]–[30]. The process phase of the APOS theory covers repeated actions and reflections to alter from being dependent on external clues to internal clues [28], [31]. The object phase of the APOS theory relates to one's ability to realize that a particular process and transformation of action are considered one unity to explicitly establish the shift [28], [32]. The schema phase is a set of conception actions, processes, objects, and schema previously established, synthesized to form a mathematics structure for solving mathematics problems [28]. The APOS theory is aimed to find out what likely to occur in a person's mind when he is about to learn the mathematics concept to find out the success and failure a person experienced when completing the mathematics tasks.

Several experts have shown some studies related to critical thinking skills. [33] have concluded that teachers were unaware of the prerequisite necessary for developing the students' critical thinking skills. Meanwhile, [34] found out that the critical thinking skill of the first-semester university students is still in the mid-level scale. [35] stated that the factor which has caused the biggest obstacle in developing the critical thinking skill is within the students themselves. [36] added that the students were difficult to develop their critical thinking skills because they have problems analyzing the possible errors and overcoming the mental impairment when they face the impairments of answers they have in geometry tasks. [37] said that the degrees of critical thinking skill viewed from the gender and Participants or majors of study in the university level have varied indicators. Based on Watson-Glaser, five variables are related to the views: inference, assumptions, deduction, interpretation, and argumentation. Another study by (Moeti et al., 2016) found out that university students possess lower description ability in their critical thinking skill since most of them did not implement the critical thinking skill they have during the training.

The studies have revealed that the critical thinking skills of students, university students, and teachers have not yet been developed optimally. However, the studies concerning the factors that cause the critical thinking skills they have are still not optimally designed or even low have never been taken. This has led the writer to study in this area to uncover and identify the errors made by the mathematics department's university students in completing the tasks on the cases of critical thinking skills of two variables linear equations using APOS theory. This research aims to see students' errors in solving issues of critical thinking skills from two variables of linear equation (TVLE). APOS theory was chosen because it can elaborate on how students' mental activities construct mathematical concepts. This initial identification is considered necessary to determine students' critical thinking skills so that appropriate learning models can be designed based on the research findings.

2. METHOD

Research Design

The study employs qualitative research using a descriptive exploratory approach. This approach obtains verbal data in student expressions when students complete a written test in a Math problem. This research design emphasizes the use of open questions and investigation. It provides opportunities for the research Participant to respond using his answer [38]. [39] argues that researchers use a qualitative approach to understand the function of a study in discussing the problem in question. In this study, the issue discussed is about students' errors in solving mathematical critical thinking skills.

Participant

All the participants are whole students in the first semester of the mathematics education department, a private university in Indonesia. The participant will be selected using the purposive sampling technique because not all the students will be chosen as a participant. Forty students had learned the material system of two-variable linear equations. Forty students voluntarily requested to solve six mathematical problems that can explore students critical thinking skills in the allotted time. All the answers sheet will be collected and checked based on critical thinking indicators. If students' answers were excellent, they would not be chosen as the participant. On the other hand, if the students' answer sheet were potentially made a mistake in solving problems related to two variables linear equations materials (TVLE), they highly potentially would be taken as a participant. Participants are strongly encouraged to have good communication skill so that the data responding to critical thinking skills will be easily explored and discovered.

Research Instruments

The research instrument employed in this study includes a test and interviews. The primary purpose of the test was to discover various mistakes made by students. The test given contained six problems that can explore students' mathematical critical thinking skills. All the problems made in daily stories form. Hopefully, students will be encouraged to find out the critical information before they respond to the problems. The entire problems had fulfilled six critical thinking indicators based on Enis, which covered focus, reason, inference, situation, clarity, and overview.

The interview purposes were to explore further information based on students' written sheet. The interview format was an unstructured interview, in which the researcher did not plan the questions to be asked to the participant. The interview process will be taken in a comfortable condition to be easily encouraged to communicate without pressure. Hopefully, students revealed the reasons related to their answers. Interview questions are based on the answer sheets of students who are being interviewed. The researcher confirmed the students interviewed about the answers to the questions that had been done.

Data Analysis

Initially, 40 university students were assigned to do a test which consisted of 6 questions in the mathematic critical thinking skill in the form of an essay. They were given 60 minutes to complete the test and submitted the answer sheets once the time was over. The next step was that the researcher selected one of the students' answers based on the assumptions of errors the student has made. This was discussed further in the discussion section of this study. Once the selection of the research Participant has done, an interview was held to confirm further responses from the student's written answer. The interview took place for 45 to 60 minutes. The interview was recorded and transcribed. The interview of this study was done based on the APOS theory to find out the errors the student has made in completing the task of critical thinking skill in mathematics cases. The researcher then analyzed the students' works independently. The following step was to triangulate the data by comparing the written test results on the critical thinking skills of mathematics cases to the development of the interview the researcher has had with the student. Afterwards, the researcher took a conclusion from the results of the triangulation.

3. RESULT AND DISCUSSION

This section will be discussed the result of the study. Further, the test results and interviews on the student's critical thinking skills were presented based on Ennis' six indicators of critical thinking skills [18].

Table 1. Student's answer on indicator focus

Student answers in Indonesian	Answer students in English
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<p>Informasi yang harus diketahui terlebih dahulu adalah berapa pasang sepatu dan 1 pasang sandal</p> $2sp + 3s = 300.000 \quad (1) \times 2$ $4sp + 6s = 600.000$ $3sp + s = 425.000 \quad (2) \times 6$ $18sp + 6s = 2.550.000$ <hr/> $14sp = 1.950.000$ $sp = 139.300$ $2sp + 3s = 300.000 \quad (1) \times 3$ $6sp + 9s = 900.000$ $6sp + 2s = 850.000$ <hr/> $6s = 250.000$ $s = 41.600$ <p>Jadi 1 pasang sepatu = 139.300 2 pasang sandal = 83.200 Jadi yang harus dibayar adalah sebesar Rp. 222.500</p>	<p>The information to know in advance is how many pairs of 1 shoes and 1 pair of slippers</p> $2sp + 3s = 300.000 \quad (1) \times 2$ $4sp + 6s = 600.000$ $3sp + s = 425.000 \quad (2) \times 6$ $18sp + 6s = 2.550.000$ <hr/> $14sp = 1.950.000$ $sp = 139.300$ $2sp + 3s = 300.000 \quad (1) \times 3$ $6sp + 9s = 900.000$ $6sp + 2s = 850.000$ <hr/> $6s = 250.000$ $s = 41.600$ <p>So 1 pair of shoes = 139.300 2 pair of slipper = 83.200 So what to pay is Rp 222.500</p>
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Students do not give the information related to the problems

The test results showed that in completing the task, the student had completed the whole task, which means that he did not comprehend the task that is he was working on. The incomprehensiveness of the student caused this to understand the indicators of the critical thinking skill. It was assumed to be the reason why he could not comprehend the questions given in the task. Consequently, an interview was held to identify the student's errors in completing the mission of critical thinking in mathematics cases by using APOS theory.

The interview results informed that the student was distracted by the questions to finish the answer of a particular question ultimately. This has caused them to have incorrect interpretations of that specific question.

Table 2. Student's answer on indicator reason

Student answers in Indonesian	Answer students in English
$2) \begin{cases} 3x - 2y = 2 & \times -2 & -6x + 4y = -4 \\ 6x - 4y = 8 & \times -1 & -6x + 4y = -8 \end{cases}$ <hr/> $-12x = 4$ $x = -\frac{1}{12} = -\frac{1}{3}$ $\therefore \text{HP} = \left\{ -\frac{1}{3}, -\frac{3}{2} \right\}$	$\begin{cases} 3x - 2y = 2 & \times -2 & -6x + 4y = -4 \\ 6x - 4y = 8 & \times -1 & -6x + 4y = -8 \end{cases}$ <hr/> $-12x = 4$ $x = -\frac{4}{12} = -\frac{1}{3}$ $3x - 2y = 2$ $3\left(-\frac{1}{3}\right) - 2y = 2$ $-1 - 2y = 2$ $-2y = 2 + 1$ $y = -\frac{3}{2}$ <p>So HP = $\left\{ -\frac{1}{3}, -\frac{3}{2} \right\}$</p>

Students do not complete the solution with explanation every step

This result has evidenced that the student only completed the task without providing any arguments from the questions. A lot of reasons or arguments were needed to make it well and correctly answered. It has shown that the student has failed to fulfil the indicator of sense in critical thinking skills. An assumption was drawn that the student has made errors in identifying the questions of the task. As a result, an interview was held based on the APOS theory held to determine the student's mistakes.

The interview results revealed that the student has made errors in scripting the TVLE presented to interpret errors. This has led them to make technical errors in calculating the TVLE. Besides, when he was asked about the concept of TVLE, which does not have any solutions, the student has failed to answer that has caused him to make conceptual errors. Other than that, when he was assigned to complete the task using a graph, he drew the diagram of TVLE, but he could not manage to determine the solutions of the graph. It was evidenced that this particular student has failed in understanding the concept of TVLE, which does not have any solutions.

Table 3. Student's answer on indicator inference

Student answers in Indonesian	Answer students in English
<p>3. harga kertas regular dan kertas mengkilat tidak sama karena harga 1 kertas regular lebih mahal dari harga kertas mengkilat</p>	<p>The price of regular and glossy paper is not the same because the price of a regular paper is more expensive than the price of glossy paper.</p>

The answers does not represent the inferences terkait dengan soal yang diberikan.

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The student's answer has shown that he was only drawing the conclusion and argumentation of the questions in the task without giving any pieces of evidence, so that it has led him to incorrect conclusions and argumentation. These have caused the student not to fulfill the indicator of inference in the critical thinking skill. An assumption has risen that the student made the errors in completing the task; therefore, an interview based on the APOS theory was conducted to identify the student's mistakes in achieving the critical thinking skill of mathematics cases.

The interview results informed that the student could not comprehend the purpose of the questions to make the incorrect interpretation. Besides, when he was given a question about the concept of TVLE, which has many solutions, he could not answer. It can be assumed that the student has made conceptual errors. Another evidence can also be seen from how the student provides a graph of the TVLE and cannot determine any solutions for the graph he has completed. These have proven that the student could not understand the concepts of TVLE, which has many answers.

Table 4. Student's answer on indicator situation

Student answers in Indonesian	Answer students in English
<p>4. SPLDV:</p> $\begin{array}{r l} x - y = 7 & \times 3 3x - 3y = 21 \\ 3x - 3y = 21 & \times 1 3x - 3y = 21 \\ \hline & 0 \end{array}$ $\begin{array}{r l} x - y = 7 & \times 1 x - y = 7 \\ 3x - 3y = 21 & \times 3 9x - 3y = 63 \\ \hline & 62 = 7 \end{array}$ <p>Jawaban yang paling benar punya Alvin nilai $(x,y) = \{7,0\}$</p> <p style="color: red; text-align: center;"><i>The written answers is quite simple, students does not adjust the answer with problem situations.</i></p>	<p>SPLDV:</p> $\begin{array}{r l} x - y = 7 & \times 3 3x - 3y = 21 \\ 3x - 3y = 21 & \times 1 3x - 3y = 21 \\ \hline & 0 \end{array}$ $\begin{array}{r l} x - y = 7 & \times 1 x - y = 7 \\ 3x - 3y = 21 & \times 3 9x - 3y = 63 \\ \hline & 62 = 7 \end{array}$ <p>he most correct answer has an Alvin value $(x,y) = \{7,0\}$</p>

These have proven that the student only provided evidence for one particular question in the task and was Alvin's proven to be the correct answer. However, the student has neglected the situation in the question, not to provide the exact arguments or reasons why Alvin's was the correct answer. This showed that the student could not fulfil the indicator of critical thinking skill and was assumed that they had made errors in completing the task. An interview based on the APOS theory was conducted to identify the student's errors in completing the task on critical thinking skills in mathematics on TVLE.

The interview results informed that the student had made procedural errors. These errors took place only when the student was only able to prove one answer to the given questions so that the answer of Alvin has become the correct answer. The student has also made technical errors in doing the division and calculation. Besides, when presented with questions about types of solutions in mathematics, he could only answer a small part of it. This has caused him to make conceptual errors.

Table 5. Student's answer on indicator clarity

Student answers in Indonesian	Answer students in English
<p>5. $x - y = -1$ $\times 2$ $2x - 2y = -2$ $2x - 2y = -2$ $\times 1$ $2x - 2y = -2$ \hline 0</p> <p>Jawaban badu dan dodi salah karena nilai $(x,y) = \{0,0\}$</p> <p style="color: red; text-align: center;"><i>Students did not clarify his answers completely</i></p>	$\begin{array}{r l} x - y = -1 & \times 2 2x - 2y = -2 \\ 2x - 2y = -2 & \times 1 2x - 2y = -2 \\ \hline & 0 \end{array}$ <p>Badu and Dodi's answer is wrong because of the value $(x,y) = (0,0)$</p>

These results have proven that Badu and Dodi's answers were using elimination. However, the operations done were incorrect and led to assuming that Badu and Dodi's answers were incorrect. Moreover, the students were unable to provide any alternative solutions to the question. This has caused the students to be unable to fulfill the indicator of clarity in the critical thinking skill. An interview on it was conducted based on the assumption that the students have made errors in completing the task. The interview done was based on the APOS theory.

The interview has informed that the students have made conceptual errors since they assumed that 0 is a solution. In another case, they also did misinterpret in concluding the particular question. When they were asked about the concept they have employed in solving the problem, they answered that they had used elimination.

Table 6. Student's answer on indicator overview/mengulang kembali

Student answers in Indonesian	Answer students in English
<p><i>Students do not interpret the answer completely related to the problems</i></p>	

This response has led to the evidence that the students were unable to correctly fix the question, so that it has caused some errors in the steps of completing the problems. It showed that the students could not manage to fulfil the overview indicator in the critical thinking skill. To identify the students' errors in completing the problems, an interview was held to respond to the assumption that the students have made errors in completing the task. The theory of APOS was implemented in the discussion.

The interview results informed that the students misinterpreted due to the inability of the students to understand the purpose of the questions. Therefore, they made mistakes in concluding their answer. Besides, they also made technical errors in calculating that $\frac{3.000}{5} = 7.000$. This occurred because the students were not careful in having the calculation

The student selected as the participant of the study was the one who had low critical thinking skills in mathematics. This particular student could not complete the six questions given in the task according to the six indicators of critical thinking skill. The student's inability to achieve critical thinking in mathematics cases was due to the student's unfamiliarity with this type of task. The student was familiar only with the kind of questions which do not demand high critical thinking skill. For this reason, it is suggested that the university students should be familiarised with the tasks which demand them to employ their critical thinking skill [40], [41]. A precise strategy is required to develop the student's critical thinking skills. One way was using the teaching model, which required the students to be active learners, the material delivery focused on results, and the problems or questions that provide challenges for the students. This statement in line with the research done by [42] as he stated that several ways to develop the critical thinking skill in mathematics in the secondary or tertiary levels of education could be taken through the implementation of a teaching strategy that demanded the students to be active learners, focusing the learning on the process on the goals, and having assessment technique which provides students with intellectual challenges rather than memorization. Besides, the students must also be supplied with metacognitive guidance to improve their critical thinking skills. As the research was done by [43], [44], metacognitive guidance in the learning environment can effectively develop investigating and critical thinking skills for students. In addition, [45] added that providing direct feedback to the students in the form of guidance enables them to discover their answers to learn more effectively.

The critical thinking skills of the first semester students in the university, which was still low, have caused errors in completing the task related to essential skills of thinking on mathematics [46], [47]. The students' common mistakes in completing critical thinking skills in mathematics can be categorized into four errors. These errors comprised the errors of interpretation, the errors of conception, the errors of procedure, and the errors of technical things. The mistake in interpretation found in the students was that the students

failed to understand the question or problems in a particular mathematics task. They were unable to interpret the inquiry into the mathematics notions. One example can be seen in how the student transcript two variables linear equation (TVLE). They misinterpreted that $\frac{8}{0}$ has the solution of (0, 0), whereas in fact, $\frac{8}{0}$ means has no resolution. On another occasion, the students were failed in drawing the conclusion required for the task. Consequently, they made mistakes in providing the correct answer to the question. The errors in clarification occurred when the student has to have an incomplete understanding of a particular concept. As stated by [48], the students failed to define or interpret two-dimensional figures due to the incomplete understanding they have of the topic.

The conceptual errors made by the students include the student's inability to respond to the questions about the indicators of TVLE, which does not have any solutions, and of the TVLE, which has many answers. In addition, the students were also unable to determine the solution using a graph. This was in line with the research done by [49], which revealed that when a person could not understand diagrams and graphs, he could not solve it since the chart. The graph carried the relevant mathematics information in the desired form. [50] They have added that understanding two variables graph is not an easy thing for university students when it is related to the schema structure on how the students completed a particular task. Therefore, the students must understand how to draw a graph of TVLE and interpret it in the form of a distinctive representation so that the conceptual errors will not occur within the students. As [51] has mentioned, the students declared to find different forms of representation which can assist them to have a better understanding and result in more solutions for the problems they have to deal with. They also made conceptual errors regarding types of unions in mathematics because they have mistaken in mentioning the unions' members or elements. This was the evidence that the student's understanding of a concept in mathematics is still low. It was supported by the findings of [29] that when the students were completing particular problems in mathematics meanwhile their understanding of the concept is not adequate. It can lead them to make errors in the conception. In line with their findings, [52] stated that it would be difficult for mathematics students to learn a particular topic of mathematics without understanding the concepts related to it. Therefore, it will lead them to perform errors in completing a task. The mistakes of conception occurred when in the learning of the previous stages of education. They were unable to understand concepts in TVLE since they were only memorizing the formulas of TVLE without internalizing the concepts. This was in line with the research done by [53] that errors the students have made chiefly originated from the previous mathematics lesson they had before. They were only doing memorization of a particular formula without understanding how to use the procedure itself. This has led the students into difficulties in correctly building up a complete knowledge of a concept [8].

Some procedural errors the students made can be found in how they neglected the notions of discussion in a particular mathematics problem. Even if the operations taken by the students were already correct, they were mistaken in concluding the result given to that specific problem. Besides, they were also mistaken in mentioning the elements within a set of numbers covered in the mathematics; they only said the sets for the integer group or whole numbers. Another error the student made was that they were mistaken in doing the sets of operations in completing a task even if they have made a correct conclusion. Therefore, it is essential to employ the whole concept necessary for completing a lesson not to be mistaken in concluding. As stated [54] in his research, the students were not only using symbols or notations, but they must also be able to give correct mathematics arguments, concluding, and generalization to complete a particular mathematics problem.

In terms of technical errors, the students have made mistakes in the calculation process. For example, one found in the division operation of whole number (integer) into a fraction has caused them to be mistaken in determining the values of x and y . This was in line with [55] that most Saudi Arabian students have a common misunderstanding about fractions and the mathematics calculation that involved the fractions in it. The errors in the division of whole numbers (integer) into fractions occurred because the students were not careful in conducting the operation. Therefore, technical errors took place. Their research [56] found that frequent mistakes made by a learner in the division operation of the whole numbers (integer) were due to his carelessness in employing the procedure itself. Therefore, he needs to occupy competent basic problem-solving skills in mathematics [57]–[60]. It is necessary to prevent the reoccurrence of similar errors from taking place.

The results analyses inform that the students were still in the phase of action in the APOS theory. This happened because they do not have any mental structure of APOS, so that they faced difficulties in solving mathematics problems. [61] In his research, the students with problems applying the essential operations and their applications were due to the absence of mental structure related to process, object, and schema phases. This has proven that the students could not manipulate the application of theorem and formulas needed for a particular mathematics problem. In the analyses using APOS theory, it was found out that the students were having a limitation of understanding the system of concept related to two variables

linear equations (TVLE) as they enter the university level. In the secondary class, they only learned about TVLE with one single solution and its graph. They did not know about TVLE with multiple answers and its chart yet. The students' incapability caused the reason for the errors made by the students to internalize the information they had with TVLE. They were lack of experience in dealing with problems of TVLE, did not comprehend the materials of TVLE completely, and had a low ability of prior knowledge related to TVLE in their previous level [62]. For these reasons, the instructors or lecturers in a university need to have a precise strategy to uncover the students' visions related to the participant being learned. [63] suggested one of the strategies to deal with this phenomenon. He indicated that to ease the students' frequent errors, an open discussion needs to be held, which focused on overcoming the students' mistakes during the learning process. [64]

further stated that assessing and analyzing the misunderstanding of the homework was regarded to be an integral approach to dig out any information about the students' ways of thinking. In addition, [64] suggested that to overcome the cognitive handicaps in completing the tasks assigned in the teaching of mathematics, it is imperative for the students to set an adequate mental effort and to have optimum practices of thinking activities (thinking operations) activating the prior knowledge and prior experiences in solving the particular problem of the tasks. [65] The instructor's practices of thinking activities must be varied based on the specific student's level, especially those of critical thinking skills in which difference among the students in different grades.

These results showed that instructors or lecturers, especially the mathematics department lecturers who teach the first semester students, must train the procedural and conceptual aspect of mathematics within the lectures. These two aspects are equally important in understanding materials in mathematics [66]. These two aspects cover TVLE of one or multiple solutions with its graphs to avoid errors. The errors identification on the students' ability in completing the critical thinking skill on the cases of TVLE in mathematics problems has assisted lecturers or instructors in recognizing the weaknesses of the students they taught. This has helped them to plan their teaching activities and performance to fix the errors that occurred. The mistakes the students made were due to their low critical thinking skills in completing the mathematics tasks. [67] stated that a person who has a conception about necessary thinking skills tends to be low and will think erroneously about critical thinking itself.

Consequently, the students' ability concerning their critical thinking skills needs to be improved to prevent similar errors from re-occurring. Once the students' critical thinking has been enhanced, they will discover formulas or rules they need to complete essential thinking problems in mathematics. [68], in their research have revealed that the improved students' critical thinking skills will possibly them to notice all the aspects necessary in helping them complete the tasks of mathematics problems which enable them to discover formulas or rules

4. CONCLUSION

Based on the data analyses and the discussion of the study, it can be concluded that the participant selected for the study has low critical thinking skills in mathematics. His incapability to fulfill the indicators of focus, reason, inference, situation, clarity, and overview was proven. The reason behind his incapacity was that he made errors in completing the critical thinking cases in mathematics. Related to the APOS theory, the errors the student has made covered (1) the errors of interpretation; he was mistaken in transforming the questions into mathematics notions, and he also mistaken in drawing conclusion required for the answer of the question; (2) the conceptual errors; he made errors in completing the answer about the criteria of TVLE which has one solution and TVLE with multiple solutions. He has mistaken in determining the solutions using graphs. Also, he has mistaken in the mathematics concept about number; (3) the procedural errors; the student has mistaken in concluding due to the problem. This was because he did not regard the scope of the discussion and was mistaken in doing the steps of completing a task; (4) technical errors; the student made a mistake in the calculation which involved operations of whole number (integer) division in the form of a fraction.

Future research should develop a new teaching model to combine APOS Theory and support students' critical thinking. Based on this research, generating a new teaching model in improving students' critical thinking was fundamental. The fact showed us that students with critical thinking would quickly adapt when facing a new mathematical problem. On the other hand, students who do not have vital consideration will face many issues solving a problem. Based on previous research, we have developed a shared awareness of critical thinking for children in learning mathematics. The upcoming research also should identify how students were thinking process during solving the problems. Identifying students thinking process will help us to expand our understanding related on how students choose the particular way than another. When teachers who acknowledge students' thinking process will be easily distinguish the students' early difficulties in solving mathematical problems. As consequently, teachers can adapt the teaching delivery materials for the next meeting.



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- 2** digilib.uinsby.ac.id
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- 3** P Utami, H Bharata. "Analysis of Mathematical Critical Thinking Skill of Junior High School Students on the Two-Variable Linear Equation System", Journal of Physics: Conference Series, 2020
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