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Table-sized matrix model in fractional learning

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Table-sized matrix model in fractional learning

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Abstract. This article provides an explanation of the fractional learning model i.e. a Table-Sized Matrix model in which fractional representation and its operations are symbolized by the matrix. The Table-Sized Matrix are employed to develop problem solving capabilities as well as the area model. The Table-Sized Matrix model referred to in this article is used to develop an understanding of the fractional concept to elementary school students which can then be generalized into procedural fluency (algorithm) in solving the fractional problem and its operation.

1. Introduction

In learning fractions, students are introduced how to learn fractions with various models such as model bar and model area [1]. Bar model is a way of problem solving by modelling the mathematical problems through an approach of Concrete Pictorial Abstract (CPA) where students learn about the story associated with the real object (concrete) [2], using rectangle bar [3], [4] or the expression of the problem [5] to represent the values and relationships contained in the story problems [6], as well as describe the situation with the number (abstract) [7] as it is presented in terms of a given story [8]. On the other hand, the area model is a visualization of two fractions which students need while comparing two pieces of fractions as shown in Figure 1.

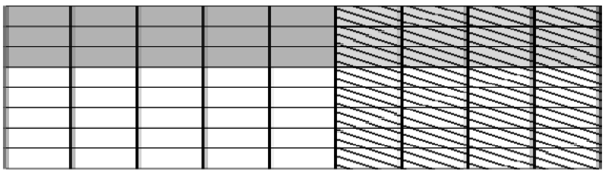

Figure 1. Area Model to compare $3/8$ and $4/9$, Source: Lamon, 2012

2. Methods

For students of Elementary School, the material of fractions always becomes a mathematical concept that is difficult to understand [9], even adults [10], because they do not have a good illustration of quantity (magnitude) of fractions [11]. Therefore, in anticipation of these difficulties, it

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needs a new innovation in the learning material of fractions, one of which is by using a Table-Sized Matrix model.

3. Result and Discussion

The originality of the idea of the Table-Sized Matrix model lies in the explanation of the size of the matrix table created and the matrix table shading in fractional operations both simple and mixed. Therefore, it can be said that Table-Sized Matrix model is an extension of Concrete Pictorial Abstract (CPA), the bar model (Singapore models) and area model. Table-Sized Matrix model is specifically used to represent the operation of two fractions with a Table-Sized Matrix so that the operation of two fractions is done by shading or coloring cells in the Table-Sized Matrix. Table-Sized Matrix model can be used to visualize a fraction with its denominations, and operations on fractions.

3.1. Fraction Table-Sized Matrix Model

Fractions are generally in the form of a/b where a is called the numerator and b is called the denominator with $b \neq 0$, $b \neq 1$. In the Table-Sized Matrix model, a is called the number of parts while b is called the number of dividers. For example, fractions $2/3$ can be visualized with 1×3 Table-Sized Matrix model, as shown in Figure 2.

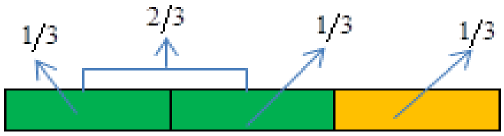

Figure 2. Table-Sized Matrix Model of 1×3

Look at Figure 2! Number 1 of the fraction $1/3$ is called the numerator or the number of parts whereas number 3 is the denominator or the number of dividers. Hence, the yellow cell in Figure 2 can be said 1 part of 3 parts or written $1/3$. While the green cells can be said 2 parts of 3 parts or written $2/3$.

Fractions with numerator numbers larger than denominator, like $7/2$, can also be represented by the Table-Sized Matrix model as shown in Figure 3 and 4. Note that the multiplication of number 2 with a number resulting a possible and approachable number 7 is $2\times 4=8$. If taken multiplication of $2\times 3=6$, it certainly does not reach number 7, so the possible model of fractional matrix table of $7/2$ is represented in Figure 3. Look at Figure 3! The Table-Sized Matrix model can be decomposed into 4 sections as seen in Figure 4.

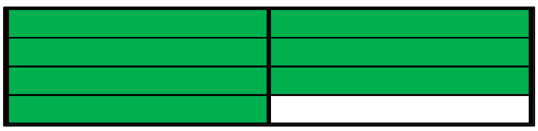

Figure 3. Table-Sized Matrix Model of 4×2

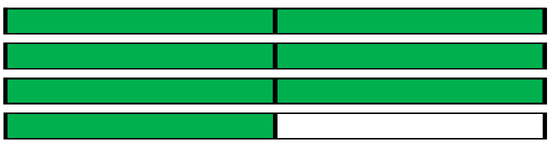

Figure 4. Table-Sized Matrix Model of 1×2 as the Representation of $7/2$

In Figure 4, there are 4 parts of the Table-Sized Matrix model of 1×2 which represents respectively $2/2, 2/2, 2/2, 1/2$. Hence, it can be written that $7/2$ is the sum of the $2/2+2/2+2/2+1/2$ or can be written as a mixed number i.e. $3\frac{1}{2}$.

3.2. *Table-Sized Matrix Model of Balanced Fraction*

Some elementary school students believe that $1/3$ smaller than $2/6$ because number 3 is smaller than number 6 and it indicates that the student has not understood the concept of fractions as a division. Therefore, Table-Sized Matrix models can help correct students' misbelief to the balanced fractions.

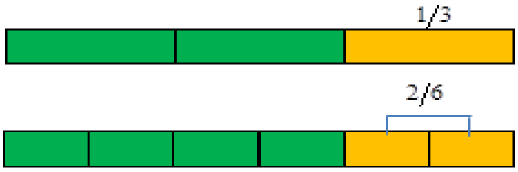

Figure 5. Table-Sized Matrix Model of 1×6

Look at Figure 5! It can be seen that $1/3$ is not smaller or bigger than $2/6$, but $1/3$ has the same value with $2/6$.

3.3. *Table-Sized Matrix Model of Fractional Operation of Summation*

The table-sized matrix model on fractional operations is determined by the denominator number of the two fractions to be operated. For example, sum $1/2$ and $1/3$, the table-sized matrix model will be formed in 2×3 where number 2 is the denominator of $1/2$ while number 3 is the denominator of $1/3$ as shown in Figure 6.

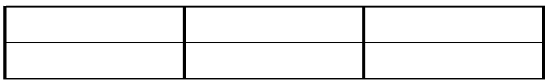

Figure 6. Table-Sized Matrix Model of 2×3

Figure 6 shows two table matrix models, namely table-sized matrix model 2×1 and 1×3 with the number of cells as much as 6 parts which further shading or coloring the table matrix model in accordance with the fractions to be added, as in Figure 7, 8 and 9. The yellow-shaded cells in Figure 7 represent the fractions of $1/2$. The green-shaded cells in Figure 8 represent the fraction of $1/3$. At once Figures 6, 7 and 8 provide an explanation of why the table-sized matrix model is named.

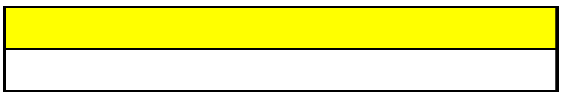

Figure 7. Table-Sized Matrix Model of 2×1 as the representation of $1/2$

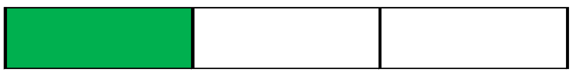

Figure 8. Table-Sized Matrix Model of 1×3 as the representation of $1/3$

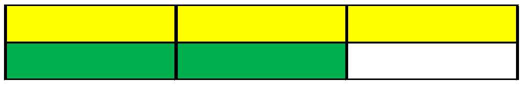

Figure 9. Table-Sized Matrix Model of 2×3 as the Summation Result of $1/2$ and $1/3$

Figure 9 is a combination of Figure 7 and 8. Figure 9 shows that one of green-shaded cells in the table-sized matrix model of 1×3 in Figure 8 is moved to another cell which hasn't been shaded. This is because the summation of two fractions adds up the whole parts. Hence, the summation result of $1/2$ and $1/3$ in Figure 9 is represented by the whole numbers of cells in yellow and green shadows, where the total sums are 5 parts of 6 parts or written in $5/6$.

Figure 10. Table-Sized Matrix Model of 5×3 as the representation of $(1+3/5)+2/3$

For fractional summation operations with numerator numbers are greater than the denominator, it can be done by converting the fractions first into mixed number. For example, the table-sized matrix that will emerge from the summation of $8/5$ and $2/3$ is 5×3. However, it is done by firstly changing $8/5$ into mixed numbers i.e. $1\frac{3}{5}$ or can be written $1+3/5$. Hence, the problem can be written $8/5+2/3=(1+3/5)+2/3$ with the representation of the matrix model as in Figure 10.

Figure 10 shows three table-sized matrix models of 5×3 which respectively from left to right represent fractions 1, $3/5$ and $2/3$, where number 1 is equivalent with $15/15$, fraction $3/5$ is equivalent $9/15$ and $2/3$ is equivalent with $10/15$. Hence, the summation results are $8/5$ and $2/3$ in Figure 10 which are represented by the total number of yellow- and green-shaded cells, where the total is 34 parts of 15 parts or written $34/15$. If it is written in symbol, the summation of $8/5$ and $2/3$ can be written in $8/5+2/3=(1+3/5)+2/3=15/15+9/15+10/15=34/15$.

3.4. Table-Sized Matrix Model of the Fractional Subtractive Operation

In fractional subtractive operation, the table-sized matrix model representation is similar to the fractional summation operation but different in shading or coloring cells to the matrix. For example, the subtraction of $1/2$ by $1/3$, the table-sized matrix model that will be resulted is 2×3 as seen in Figure 11.

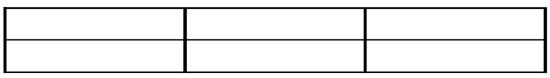

Figure 11. Table-Sized Matrix Model of 2×3

The next step is to shade or color the cells that represent fractions $1/2$ first because $1/3$ is the number to be subtracted as shown in Figure 12. Then, color the cells that represent fractions $1/3$, as seen in Figure 13. The next step is, subtract the yellow cells with green cells, as shown in Figure 14.

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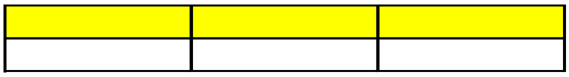

Figure 12. Table-Sized Matrix Model of 2×3 as the representation of $1/2$

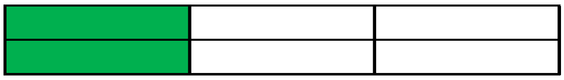

Figure 13. Table-Sized Matrix Model of 2×3 as the representation of $1/3$

Figure 14. Table-Sized Matrix Model of 2×3 as the subtraction result of $1/2$ by $1/3$

Figure 14 shows that yellow cells leave 1 cell which is the result of subtraction $1/2$ by $1/3$. Hence, the result of the subtraction of $1/2$ by $1/3$ is $1/6$ reduction.

3.5. Table-Sized Matrix Model of Fractional Multiplication Operation of Pure Number with Pure Number

In the multiplication operation of two fractions, the representation of the table-sized matrix model is similar to the summation and subtraction operations but different in shading or coloring pattern. In the multiplication, the cell slices are the results. For example, multiplication $1/4$ and $2/3$, the table-sized matrix model that will be formed is 4×3 as seen in Figure 15.

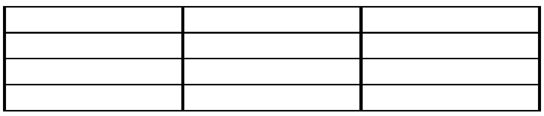

Figure 15. Table-Sized Matrix Model of 4×3

The next step is to color the cells that represent fractions $1/4$ and $2/3$ as seen in Figure 16. Figure 16 shows that the red cells are the slices of the yellow and green cells that is simultaneously the answer or multiplication result of $1/4$ and $2/3$ i.e. $2/12$ or $1/6$.

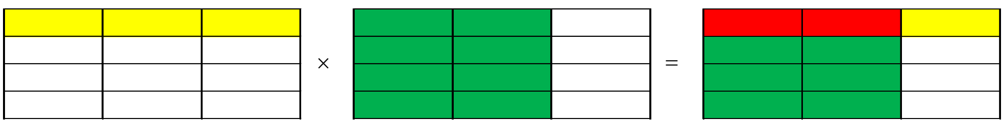

Figure 16. Table-Sized Matrix Model of 4×3 as the representation of $1/4\times2/3$

3.6. Table-Sized Matrix Model of Fractional Multiplication Operation of Pure Number and Impure Number

For fractional multiplication operations with numerator numbers greater than the denominator, it can be done by converting the fractions first into mixed numbers. For example, multiplication of $7/3$ and

$2/5$, the table-sized matrix model that will be formed is 3×5 by firstly changing the fraction $7/3$ into a mixed number as seen in Figure 17.

Figure 17. Table-Sized Matrix Model of 3×3 as the representation of $7/3$

Figure 17 shows that the mixed number of $7/3$ is $2\frac{1}{3}$ or $2+1/3$. Hence, the form of the problem becomes $(2\times 2/5)+(1/3\times 2/5)$ which is equivalent to $(2+1/3)\times 2/5$ and the table-sized matrix model is seen as in Figures 18 and 19.

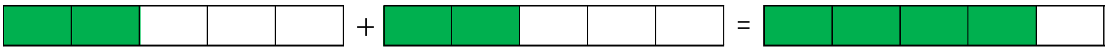

Figure 18. Table-Sized Matrix Model of 1×5 as the representation of $2\times 2/5$

From Figure 18, it can be seen that $2\times 2/5=4/5$.

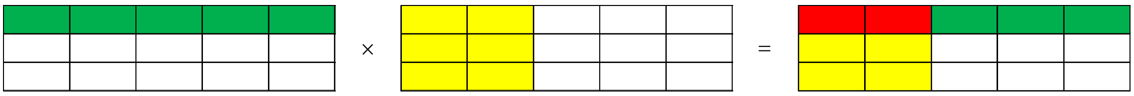

Figure 19. Table-Sized Matrix Model of 3×5 as the representation of $1/3\times 2/5$

Figure 19 shows that multiplication result of $1/3\times 2/5=2/15$ and the summation of $4/5$ and $2/15$ is represented by the table-sized matrix model as seen in Figure 20. From Figure 20, it can be concluded that the multiplication result of $7/3$ and $2/5$ is $14/15$.

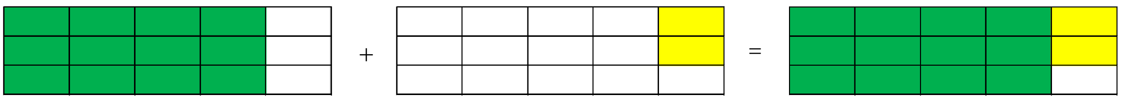

Figure 20. Table-Sized Matrix Model of 3×5 as the representation of $4/5+2/15$

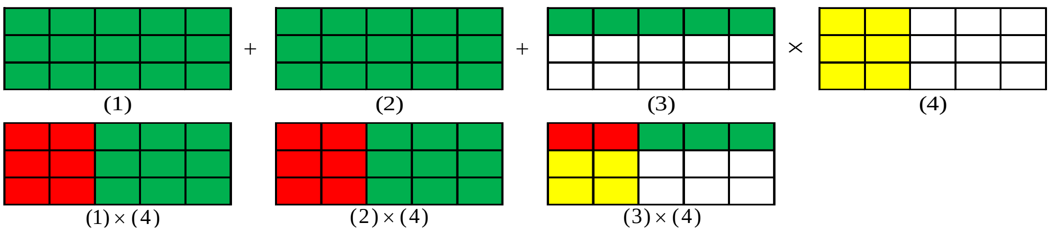

Figure 21. Table-Sized Matrix Model of 3×5 as the representation of $(2+1/3)\times 2/5$

In other words, the multiplication $7/3$ and $2/5$ can be done by making table-sized matrix model of 3×5 by firstly converting fraction of $7/3$ to be the mixed number of $2\frac{1}{3}$ or $2+1/3$ as seen in Figure 21. Figure 21 also shows that the multiplication result of $7/3$ and $2/5$ is $14/15$ which is resulted by multiplying the model $(1)\times(4)$, $(2)\times(4)$ and $(3)\times(4)$ where the multiplications among the matrix models have 14 red cell slices in total.

3.7. Table-Sized Matrix Model of Fractional Division Operation of Pure Number and Pure Number

In the division operation of two fractions, the matrix model table representation is similar to other operations but different in shading or color patterns and different ways of reading the shading or color of the matrix. For example, division of $3/4$ by $1/3$, the table-sized matrix model that will be formed is 4×3 as seen in Figure 22.

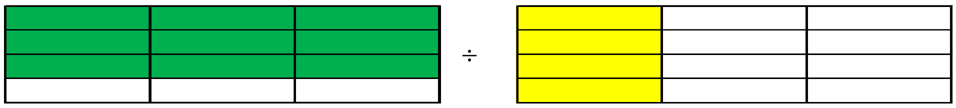

Figure 22. Table-Sized Matrix Model of 4×3 as the representation of $3/4\div1/3$

From Figure 22, it can be seen that $3/4\div1/3=9/4$ where number 9 is obtained from the number of green cells while number 4 is obtained from the number of yellow cells.

3.8. Table-Sized Matrix Model of Fractional Division Operation of Impure Number and Pure Number

In fractional division operation of impure and pure number, the table-sized matrix model representation is similar to other operations but different in shading or color patterns and different ways of reading the shading or color of the matrix. For example, division $4/3$ by $1/2$ should firstly change $4/3$ into the mixed number of $1\tfrac{1}{3}$ or $1+1/3$. By doing so, the mathematical problem becomes $4/3\div1/2=(1+1/3)\div1/2$ with the table-sized matrix model that will be formed is as seen in Figure 23.

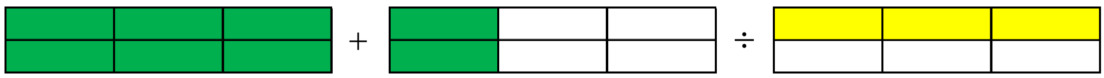

Figure 23. Table-Sized Matrix Model of 2×3 as the representation of $(1+1/3)\div1/2$

From Figure 23, it can be seen that the division result of $4/3$ by $1/2$ is $8/3$ where number 8 is obtained from the number of green cells while number 3 is obtained from the number of yellow cells.

3.9. Table-Sized Matrix Model of Fractional Division Operation of Impure Number and Impure Number.

In fractional division operation of impure and impure number, the table-sized matrix model representation is similar to other operations but different in shading or color patterns and different ways of reading the shading or color of the matrix. For example, division $5/2$ by $4/3$ should firstly change $5/2$ and $4/3$ into the mixed number of $2\tfrac{1}{2}$ or $2+1/2$ and $1\tfrac{1}{3}$ or $1+1/3$. By doing so, the mathematical problem becomes $5/2\div4/3=(2+1/2)\div(1+1/3)$ with the table-sized matrix model that will be formed is 2×3 as seen in Figure 24.

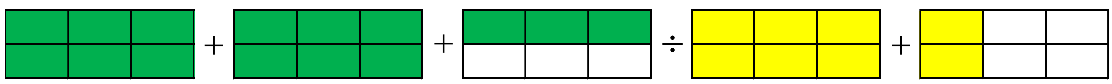

Figure 24. Table-Sized Matrix Model of 2×3 as the representation of $(2+1/2)\div(1+1/3)$

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From Figure 24, it can be seen that the division result of $5/2$ by $4/3$ is $15/8$ where number 15 is obtained from the number of green cells while number 8 is obtained from the number of yellow cells.

4. Conclusion

Based on the description above, the learning of fractional materials with table-sized matrix models is expected to help students visualize and model kinds of fractions, balanced fractions and fractional operations which can further enhance the understanding of the concept of fractions, balanced fractions and fractional operations. Improved understanding of the concept of fractions through the table-sized matrix model is expected to help students improve procedural fluency in fractional operation.

Dealing with the table-sized matrix model in learning fractional materials, further research is needed to ascertain whether the matrix model can be understood and followed by students and teachers and whether it can improve the conceptual and procedural understanding (algorithm) of elementary school students' fractions.

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