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Some cycle-supermagic labelings of the calendula graphs

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Abstract. In this paper, we introduce a calendula graph, denoted by $Cl_{m,n}$. It is a graph constructed from a cycle on m vertices C_m and m copies of C_n which are $C_{n_1}, C_{n_2}, \cdots, C_{n_m}$ and grafting the i-th edge of C_m to an edge of C_{n_i} for each $i \in \{1, 2, \cdots, m\}$. A graph $G = (V, E)$ admits a C_n-covering, if every edge $e \in E(G)$ belongs to a subgraph of G isomorphic to C_n. The graph G is called cycle-magic, if there exists a total labeling $\varphi : V \cup E \to \{1, 2, \ldots, |V| + |E|\}$ such that for every subgraph $C_n' = (V', E')$ of G isomorphic to C_n has the same weight. In this case, the weight of C_n, denoted by $\varphi(C_n')$, is defined as $\sum_{v \in V(C'_n)} \varphi(v) + \sum_{e \in E(C'_n)} \varphi(e)$. Furthermore, G is called cycle–supermagic, if $\varphi : V \to \{1, 2, \ldots, |V|\}$. In this paper, we provide some cycle-supermagic labelings of calendula graphs. In order to prove it, we develop a technique, to make a partition of a multiset into m sub-multisets with the same cardinality such that the sum of all elements of each sub-multiset is same. The technique is called an m-balanced multiset.

1. Introduction

The graphs considered here are finite, undirected, and simple. The vertex set and the edge set of a graph G are denoted by $V(G)$ and $E(G)$, respectively. An H-(super)magic labeling was first studied by Gutiérrez and Lladó in 2005 [3]. Lladó and Moragas [5] studied some cycle-(super)magic behavior of several classes of connected graphs. They gave several families of C_r -magic graphs for every $r \geq 3$. Maryati et al. [10] contributed to C_n-supermagic labelings of c copies of C_n. Some other results on C_n-supermagic labelings of several classes of graphs can be found in [1, 2, 6, 7, 8, 11,12, 13, 14].

This paper is organized as follows. In section 2, we define a new class of graph that we call a calendula graph. It is inspired by comb product graph [4, 14]. In section 3, we develop the concept of an m-balanced multiset [8]. It is a technique to partition a multiset to obtain m submultisets such that each submultiset has the same cardinality and the sum of all elements in each submultiset has a same value. This result is used to prove our main result. In the last section, we study C_n-supermagic labelings of calendula graphs. In this paper, we use the notation $[a, b]$ to mean $\{a \leq x \leq b \mid a, b \in \mathbb{Z}^+\}$ and $\sum A$ to mean $\sum_{a \in A} a$. We define $\{a\} \uplus \{a, b\} = \{a, a, b\}$.

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2. Calendula Graphs

Let $m \geq 3$ and $n \geq 3$. Let C_n be a cycle on m vertices. A *calendula graph*, denoted by $Cl_{m,n}$, is a graph constructed from C_m and m copies of C_n which are $C_{n_1}, C_{n_2}, \cdots, C_{n_m}$ and grafting the i-th edge of C_m to an edge of C_{n_i} for each $i \in \{1, 2, \cdots, m\}$. For illustration, we can see $Cl_{6,4}$ in figure 1.

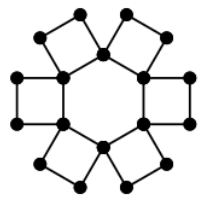

Figure 1. A calendula graph $Cl_{6,4}$.

We can check that the order of $Cl_{m,n}$ is $m(n-1)$ and the measure of $Cl_{m,n}$ is mn. It mean that $\left|V\left(Cl_{m,n}\right)\right| + \left|E\left(Cl_{m,n}\right)\right| = m(2n-1)$. For $m \neq n$, $Cl_{m,n}$ has m subgraph C_n' which isomorphic to C_n. As for $m = n$, $Cl_{m,n}$ has $(m + 1)$ subgraph C_n' which isomorphic to C_n. We can also check that $Cl_{m,n}$ contains C_n-covering. Let the vertex set and the edge of $Cl_{m,n}$, respectively, be as follows:

$$V\left(Cl_{m,n}\right) = \left\{v_i^j \mid i \in [1, m] \text{ and } j \in [1, (n-1)]\right\} \text{ and}$$

$$E\left(Cl_{m,n}\right) = \left\{e_i^j \mid i \in [1, m] \text{ and } j \in [1, n]\right\}.$$

3. *m*-Balanced Multiset

A *multiset* is a set which allows the same elements. Let a multiset $V = \{a_1, a_2, ..., a_m\}$ and a multiset $W = \{a_1, a_2, ..., a_n\}$. Define $V \uplus W = \{a_1, a_2, ..., a_m, a_1, a_2, ..., a_n\}$. An m-balanced multiset defined as follows. Let $m \in \mathbb{Z}^+$ and Y is a multiset of positive integers. Y is called *m-balanced*, if there are m submultiset of Y, that is $Y_1, Y_2, \ldots, Y_m$, such that for each $i \in [1, m]$, satisfies $|Y_i| = |Y| m^{-1}$, $\sum Y_i = \left(\sum Y\right) m^{-1} \in \mathbb{Z}^+$, and $\uplus_{i=1}^m Y_i = Y$. We need the next lemma to prove that $Cl_{m,n}$ is C_n-supermagic.

Lemma 1. Let m and n are positive integers with $m \geq 3$ and $n \geq 3$. If a multiset $X = [1, m] \uplus [1, m(2n-1)]$, then X is m-balanced.

Proof.

For each $i \in [1, m]$ and $j \in [1, 2n]$, define a multiset $X_i = \{a_{i,1}, a_{i,2}, ..., a_{i,2n}\}$ with

$$a_{i,j} = \begin{cases} i, & \text{for } i \in [1, m] \text{ and } j = 1; \\ i + 1, & \text{for } i \in [1, m-1] \text{ and } j = 2; \\ 1, & \text{for } i = m \text{ and } j = 2; \\ 2m - i + 1, & \text{for } i \in [1, m] \text{ and } j = 3; \\ 3m - i, & \text{for } i \in [1, m-1] \text{ and } j = 4; \\ 3m, & \text{for } i = m \text{ and } j = 4; \\ (j-1)m - i + 1, & \text{for } i \in [1, m] \text{ and } j \in [5, n],\ j \equiv 0 \bmod 2; \\ (j-2)m + i, & \text{for } i \in [1, m] \text{ and } j \in [5, n],\ j \equiv 1 \bmod 2. \end{cases}$$

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Next, for every $i \in [1, m-1]$ and $j \in [1, 2n]$, we obtain

$$\sum X_i = 1+(i+1)+(2m-i+1)+(3m-i)+(3m+i)+(5m-i+1)+(5m+1)+(7m-i+1)+\ldots$$
$$+((2n-3)m+i)+((2n-1)m-i+1)$$
$$= 2m+3m+n+\sum_{t=2}^{n-1} 4mt$$
$$= 2mn^2-2mn+m+n.$$

For every $i = m$ and $j \in [1, 2n]$, we get

$$\sum X_i = m+1+(m+1)+3m+4m+(4m+1)+6m+(6m+1)+\ldots+((2n-2)m)+((2n-2)m+1)$$
$$= 2m+3m+n+\sum_{t=2}^{n-1} 4mt$$
$$= 2mn^2-2mn+m+n.$$

For $j \in [1, 2n]$, $A_j = \{a_{i,j} \mid 1 \le i \le m\}$, let

$$A_j = \begin{cases} [1, m] \text{ for } j \in [1, 2]; \\ \left[((j-2)m+1), ((j-1)m)\right] \text{ for } j \in [3, 2n]. \end{cases}$$

It can be checked that $A_1 \uplus A_2 \uplus \ldots \uplus A_{2n} = X$ and $\uplus_{i=1}^{m} X_i = X$ Additionally, for each $i \in [1, m]$, we obtain $|X_i| = 2n$ and $\sum X_i = 2mn^2-2mn+m+n$. Therefore, for $m \ge 3$ and $n \ge 3$, we get that X is m-balanced.

2 Calendula Graphs are Some Cycle-Supermagic

In this section we show that a calendula graph $Cl_{m,n}$ for any positive integers m and n with $m \ge 3$ and $n \ge 3$ is C_n-supermagic.

Theorem 2. *Let m and n be two integers with $m \ge 3$ and $n \ge 3$. Let $Cl_{m,n}$ be a calendula graph, then $Cl_{m,n}$ is C_n-supermagic.*

Proof.

Let C_n' be a subgraph of $Cl_{m,n}$ which isomorphic with C_n. Define a total labeling $\varphi : V(Cl_{m,n}) \cup E(Cl_{m,n}) \to \{1,2,\ldots,m(2n-1)\}$ as follows.

(i) Let $m \ne n$. Let a multiset $X = [1,m] \uplus [1, m(2n-1)]$. Partition X into several submultisets, X_i with $i \in [1,m]$ based on the above Lemma 1. For $i \in [1,m]$, label v_i^j and e_i^j on C_n' by using elements in X_i and the smallest label to label the vertices such that every subgraph C_n' on $Cl_{m,n}$ applies

$$\varphi(C_n') = 2mn^2-2mn+m+n.$$

Therefore, for $m \ne n$, we obtain φ is an C_n -super magic labeling on $Cl_{m,n}$.

(ii) Let $m = n$. Since $Cl_{m,n}$ has $(n+1)$ subgraphs C_n', we need a modification labeling (i) such that every subgraph C_n' has the same weight. We divide into two subcases.

(ii.a) For $m = n \equiv 0 \bmod 2$.

First, do the labeling as in (i). Furthermore, re-do the labeling on some edge e_i^j by swapping a pair of edge label e_i^j which are at the same C_n' using the following way:

- exchange thelabel edge e_i^1 with e_i^n, for $i = 1$;
- exchange the label edge e_i^1 with e_i^{n-1}, for $n > 4$, $i \in \left[2, \left(\frac{1}{2}n - 1\right)\right]$ and $i \in \left[\left(\frac{1}{2}n + 1\right), n\right]$;
- exchange the label edge e_i^1 with e_i^{n-2}, for $n > 4$, $i = \frac{1}{2}n$, and for $n = 4$, $i = 2$.

This re-labeling does not change the weight of $n$subgraphC_n' which is obtained on (i). Furthermore, it is obtainedlabelling of a new subgraphC_n' with equal weight such that there are $(n + 1)$ subgraphC_n' which has same weight on $Cl_{m,n}$.

(ii.b) For $m = n \equiv 1 \bmod 2$.

Do labeling as in (i). Furthermore, do re-labeling on some edge e_i^j by swapping a pair of label edge e_i^jwhich are at the sameC_n' in the following way:

- exchange the label edge e_i^1 with $e_i^{\frac{1}{2}(n+1)}$, for $i = 1, n$;
- exchange the label edge e_i^1 with e_i^n, for $i \in [2, (n - 1)]$.

Similarly to (ii.a), this re-labeling does not change the weight of $n$subgraphC_n' which is obtained in (i). Furthermore, it is obtained labeling a new subgraphC_n' with equal weight such that there are $(n+1)$ subgraphC_n'which has same weight on $Cl_{m,n}$ for $m = n \equiv 1 \bmod 2$.

From (i), (ii.a), and (ii.b), we conclude that $Cl_{m,n}$ is C_n -supermagic for any integers m and n with $m \geq 3$ and $n \geq 3$. □

For illustration, in figure 2, figure 3, and figure 4 we show cycle-supermagic labelings on calendula graphs$Cl_{6,4}$,$Cl_{4,4}$, and$Cl_{5,5}$, respectively.

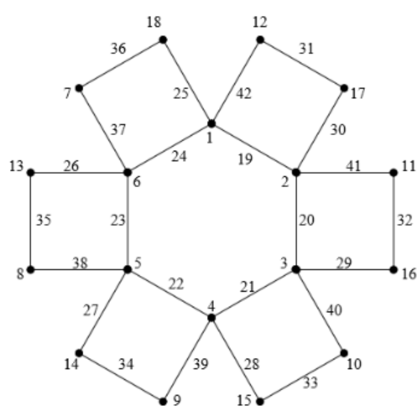

Figure 2. C_4–supermagiclabeling on $Cl_{6,4}$graph.

In figure2, it can be checked that the number of labels of each $C_4$is constant. We obtain the weight of 6 subgraphsC_4as follows.

- $\varphi\left(C_4^1\right) = 1 + 2 + 12 + 17 + 19 + 30 + 31 + 42 = 154$
- $\varphi\left(C_4^2\right) = 2 + 3 + 11 + 16 + 20 + 29 + 32 + 41 = 154$
- $\varphi\left(C_4^3\right) = 3 + 4 + 10 + 15 + 21 + 28 + 33 + 40 = 154$

- $\varphi\left(C_4^4\right)=4+5+9+14+22+27+34+39=154$
- $\varphi\left(C_4^5\right)=5+6+8+13+23+26+35+38=154$
- $\varphi\left(C_4^6\right)=6+1+7+18+24+25+36+37=154$.

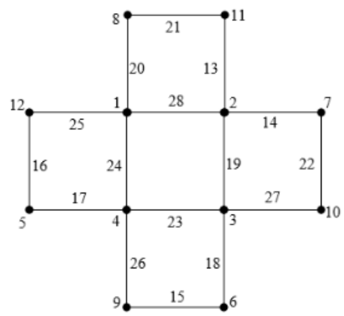

Figure 3. C_4 – supermagic labeling on $Cl_{4,4}$ graph.

In figure3, it can be checked that the number of labelsof each C_4 is constant. We obtain the weight of 5 subgraphs C_4 as follows.

- $\varphi\left(C_4^1\right)=1+2+8+11+13+20+21+28=104$
- $\varphi\left(C_4^2\right)=2+3+7+10+14+19+22+27=104$
- $\varphi\left(C_4^3\right)=3+4+6+9+15+18+23+26=104$
- $\varphi\left(C_4^4\right)=4+1+5+12+16+17+24+25=104$
- $\varphi\left(C_4^5\right)=1+2+3+4+19+23+24+28=104$

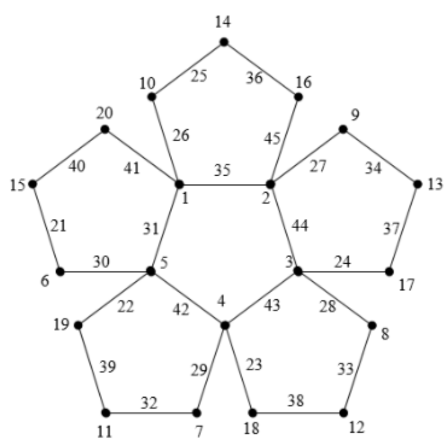

Figure 4. C_5 –supermagiclabeling of $Cl_{5,5}$graph

In figure 4, it can be checked that the number of labelsof each$C_5$is constant. We obtain the weight of 6sub graphs C_4 as follows.

- $\varphi\left(C_5^1\right)=1+2+10+14+16+25+26+35+36+45=210$
- $\varphi\left(C_5^2\right)=2+3+9+13+17+24+27+34+37+44=210$
- $\varphi\left(C_5^3\right)=3+4+8+12+18+23+28+33+38+43=210$
- $\varphi\left(C_5^4\right)=4+5+7+11+19+22+29+32+39+42=210$
- $\varphi\left(C_5^5\right)=5+1+6+15+20+21+30+31+40+41=210$
- $\varphi\left(C_5^1\right)=1+2+3+4+5+31+35+42+43+44=210$

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