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Enhancement method for assignments of staff in multiple events

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Abstract. A problem of staff assignment in an organization with multiple assignments of scheduled events is addressed here. The objective is to propose an efficient and practical approach for enhancing a given policy of staff assignments to attain a more efficient operation. To address the problem, a multi-period assignment model formulation is employed here. Using this approach, an efficient heuristic method is devised to obtain enhancements of any given solution policy. A numerical example is provided for clarity in implementing the proposed method. This research is expected to provide a meaningful contribution to managing staff assignments in various types of organizations.

1 Introduction

Staff assignments play an important role in managing resources in many organizations, especially the human resource, in order to stay efficient while being pressed for achieving their operational requirement. When an assignment of staffs occurs in multiple periods, the problem usually becomes complicated by the need to match one or more requirements simultaneously, such as budget limit, skill criteria, timing, or career demand of the employee. Staff assignments in multiple periods can be categorized as a multi-period assignment problem. Obtaining an optimal solution using exact mathematical programming for a large instance of the problem in terms or the number of personnel, positions, and events, are computationally prohibitive.

Multi-period assignment problems arise in many instances, which include organizational training programs [1], doctor assignment ([2] and [3]), in scheduling law enforcement agency to elections [4], and in assigning airline customer service agent [5]. Such problems have attracted many researchers for years in which it is formulated as a multi-dimensional assignment problem [6]. It can also be formulated as a multiperiod assignment problem ([7] and [2]).

Optimal solution methods to such a problem have been sought by several researchers, such as provided by [8], [9], [10], and [11]. Since the multi-period assignment is categorized as an NP-hard complexity ([6] and [12]), exact mathematical approaches can be computationally intractable. Efficient heuristics are then sought by many as an alternative approach for tackling large instances of the problems. Most of the heuristics are usually

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developed for a specific subclass of problems. This research enriches the literature by proposing an alternative heuristic approach for a specific staff assignments problem that arises in educational organizations that hold multiple training events, which is described in the next section.

2 Description of the problem

The assignment problem addressed here pertains to a situation in a typical training board of a government institution where several training events are planned. Employees of the organization must be assigned to choices of staff positions of training events, or referred to as events for compactness, scheduled in an annual planning horizon. Placement of an employee into a staff position of events shall accord to the requirements of the corresponding staff position, such as the employee's rank, specialization, fee, or other criteria. Hence, a weighting preference or 'cost' is applied in each staff position to provide some sort of suitability measure of assigning a person into a staff position. The less suitable a person for a staff position the higher the cost of assigning her to the position. Conversely, the more suitable the lower the cost. It is assumed that a prescribed number of assignments in the planning horizon is determined for every employee. The objective of solving an assignment problem addressed here is, therefore, to minimize the total cost of placing a set of assignments of staff in the planning horizon.

2.1 Problem formulation

To describe the mathematical problem formulation, the following symbols are employed.

- K = number of events in the planning horizon;
- L_k = number of staff position in event k;
- N = total number of person (employee) to be assigned;
- x_{ijk} = decision variable of person i assigned on j-th staff position of an event k. The value $x_{ijk} = 1$ if the corresponding person is assigned or, otherwise, $x_{ijk} = 0$ if none is assigned;
- a_{ijk} = 'cost' of assigning person i into position j of k-th event;
- b_i = predetermined total number of assignment for person i into all scheduled events.

Since the objective of the problem addressed here is to minimize the total cost incured by assigning the employees into staf position, the following objective function is defined as.

$$Z = \sum_{k=1}^{K} \sum_{j=1}^{L_k} \sum_{i=1}^{N} a_{ijk} x_{ijk} \quad (1)$$

Note that (a) each person can only be placed in exactly one staff position in an event; and (b) in each staff position of an event there must be assigned exactly one person for the position. Hence, the assignment problem is defined as follows. Find x_{ijk} for $i = 1, \ldots, N$; $j = 1, \ldots, L_k$; and $k = 1, \ldots, K$ so as to minimize Z subject to

$$\sum_{j=1}^{L_k} x_{ijk} \leq 1 \text{ for } x_{ijk} \leq 1\text{; for } k = 1, \ldots, K\text{; and for } i = 1, \ldots, N, \quad (2)$$

$$\sum_{i=1}^{N} x_{ijk} = 1 \text{ for } j = 1, \ldots, L_k \text{ and for } k = 1, \ldots, K, \quad (3)$$

$$\sum_{k=1}^{K} \sum_{j=1}^{L_k} x_{ijk} = b_i \text{ for } i = 1, \ldots, N, \quad (4)$$

$$x_{ijk} = \{0, 1\} \ \forall\ i, j, k \quad (5)$$

where

$$x_{ijk} = \begin{cases} 1 & \text{if person } i \text{ is assigned to staff } j \text{ position at event } k \\ 0 & \text{if none is assigned to staff } j \text{ position at event } k \end{cases} \quad (6)$$

2.2 Constraints and computational complexity

The first constraint (2) assures a person cannot be assigned to more than one staff position in each event. The second constraint (3) assures that each staff position in an event must be assigned a person to it, i.e., a blank staff position is not allowed. The constraints (4) is the pre-determined total number of assignment of each person in the planning horizon. In this formulation, it is assumed the constraints (4) should be binding. The last constraint (5) assures the decision variables take binary value.

The problem presented in (1) – (5) above can be categorized as a special case to a Multi-Period Assignment Problem. Specifically, it is of the three-dimensional assignment problem, in which the problem is of an NP-Hard complexity. A solution to the problem using an exact mathematical approach, such as Branch and Bound can be computationally intractable for a large size of instances. A more efficient approximation method approach is sought in this research in order to obtain an acceptable solution within a reasonable time. The following section will describe the method.

3 Methodology for assignments and computational application

The solution method sought here takes two general steps: (i) To obtain an initial, feasible solution to a given problem; and, (ii) to perform an enhancement step to obtain a better objective value. The solution method for a feasible solution in the first step is assumed to be obtained from the work by [1]. Hence, it is focused here to develop a method for the second step, i.e., the enhancement solution, which will be described next.

3.1 Enhancement method

The enhancement method proposed here consists of two phases of iterations, in which incremental improvements of assignments are sought in each phase. In the first phase of iterations, improvements of assignments are sought within each event. Here, every pair of two assignments within the same event is investigated for possible exchange in assignment positions that will result in incremental improvement of the objective function without violating feasibility to the given constraints. In the second phase, improvements of assignments are sought between every pair of two assignments from two different events. The mechanism to obtain incremental improvement is the same as that in the first phase except that it is done between two different events. The enhancement procedure is given as follows:

1. *Phase I*. *For* each event k do the following :
   For each pair of two assigned persons within the event k, say, person c assigned to position e and person d assigned to position f, *do* :
   a. *If* $a_{cek} + a_{dfk} > a_{dek} + a_{cfk}$ then switch the assignment positions between the two persons, i.e., the person c is assigned to position f and the person d is assigned to position e.
   b. *Else*, keep the current assignment of the corresponding pair.
2. *Phase II*. *For* each pair of two different events, say, events k and l, do the following :
   For each pair of two assigned persons, say person c assigned to position e in the event k and person d assigned to position f in the event l, do :
   a. *If* $a_{cek} + a_{dfl} > a_{dek} + a_{cfl}$ then switch the assignment positions between the two persons, i.e., the person c is assigned to position f in the event l and the person d is assigned to position e in the event k.
   b. *Else*, keep the current assignment of the corresponding pair.
3. *Stop*. □

It can be observed in the foregoing procedure of the first phase there are L_k by N iterations. In the second phase, there are K by L_k by N iterations at most. Taking the largest, therefore, the computational complexity of the proposed procedure is of the cubic order polynomial complexity. The following section is provided to illustrate the application of the proposed approach to a problem instance.

3.2 Computational application

Consider a hypothetical situation where a training division is planning four events of training in the next year. There are four persons to be assigned to three staff positions in every event. Each person must be assigned three times in the given planning horizon. Hence, we have $N = 4$, L_k`s = 3, $K = 4$, and $b_i$`s = 3. A linear scale of weights 1, 2, 3, 4 and 5 is utilized to represent the costs indicating one's suitability for being assigned to a certain staff position. The value of 1 indicates the 'most suitable' and 5 indicates the 'least suitable.' Suppose the costs of assigning employees to various staff positions, i.e. the a_{ijk}`s, are shown in the following matrix (Fig. 1).

Event *k*	1			2			3			4		
Staff *j*	1	2	3	1	2	3	1	2	3	1	2	3
Person *i* = 1	4	5	1	4	4	5	3	2	4	3	5	4
2	5	4	3	2	3	1	4	1	2	4	1	1
3	5	3	1	2	3	1	4	3	5	2	1	1
4	2	2	5	4	2	2	2	3	1	5	5	1

Fig 1. Cost matrix (a_{ijk}) for staff assignment.

It is assumed that an initial assignment solution is given by applying the method given by [1]. The initial solution is given as $x_{311} = x_{221} = x_{131} = x_{112} = x_{422} = x_{232} = x_{113} = x_{323} = x_{433} = x_{414} = x_{324} = x_{234} =1$ and the remaining x_{ijk}`s are zeros. The initial solution is depicted by the following matrix in Fig 2. The corresponding objective value (1) of the initial solution is $Z = 31$. Our proposed method will be implemented next to seek an enhancement of the given foregoing initial solution.

Event *k*	1			2			3			4		
Staff *j*	1	2	3	1	2	3	1	2	3	1	2	3
Person *i* = 1	0	0	1	1	0	0	1	0	0	0	0	0
2	0	1	0	0	0	1	0	0	0	0	0	1
3	1	0	0	0	0	0	0	1	0	0	1	0
4	0	0	0	0	1	0	0	0	1	1	0	0

Fig. 2. Initial solution of the staff assignment.

Upon executing *Phase I* (within-event) of the enhancement procedure, it is found two pairs of staff assignments where switching position in each pair of the assignments yield better results. Specifically,

a) In Event 1: Since $a_{311} + a_{131} = 6 > a_{111} + a_{331} = 5$ then the position of Person 3 in Staff 1 and the position of Person 1 in Staff 3 are switched each other so that the position Person 3 is now assigned to Staff 3 and the position of Person 1 is now assigned to Staff 1. Thus, $x_{311} = x_{221} = 0$ and $x_{111} = x_{331} = 1$, yielding incremental reduction of $(a_{311} + a_{131}) - (a_{111} + a_{331}) = 1$ of the objective value.

b) In Event 4: Since $a_{414} + a_{234} = 6 > a_{214} + a_{434} = 5$, the position of Person 4 in Staff 1 and the position of Person 2 in Staff 3 are switched each other. This yield an incremental reduction of $(a_{414} + a_{234}) - (a_{214} + a_{434}) = 1$ of the objective value.

Event k	1			2			3			4		
Staff j	1	2	3	1	2	3	1	2	3	1	2	3
Person i 1	1	0	0	1	0	0	1	0	0	0	0	0
2	0	1	0	0	0	1	0	0	0	1	0	0
3	0	0	1	0	0	0	0	1	0	0	1	0
4	0	0	0	0	1	0	0	0	1	0	0	1

Fig. 3. Enhancement solution from Phase I.

Fig. 3 shows the resulting enhancement of Phase I with the corresponding objective value (1) is $Z = 29$. Upon executing the *Phase II* (between-events) on the foregoing result, there is a pair of staff assignment that can be switched to obtain an incremental enhancement. That is, the position of Person 2 in Staff 2 of the Event 1 is switched with the position of Person 4 in Staff 3 of the Event 3, which yield an incremental reduction of $(a_{221} + a_{433}) – (a_{421} + a_{233}) = 1$ of the objective value. Hence, the final solution is as $x_{111} = x_{421} = x_{231} = x_{112} = x_{422} = x_{232} = x_{113} = x_{323} = x_{233} = x_{214} = x_{324} = x_{434} =1$ and the remaining x_{ijk} 's are zeros. The corresponding objective value (1) of the final solution is $Z = 28$, which is approximately 90% of the initial objective value (improvement).

4 Concluding remarks

This research proposed an enhancement methodology for assigning staff to multiple scheduled events. The method is intended to seek an improvement of a given initial staff assignment to obtain an assignment with better objective values. It is shown that the iterative approach given here is practical and efficient. The proposed method is expected to provide a useful tool for managerial decision making that deals with large instances of staff assignment. The problem addressed here has assumed the prescribed number of assignment of each employee is translated into binding constraints, as shown in (4). To provide a more adaptable model in representing many real situations, however, the assumption of binding formulation shall be relaxed. Hence, further research can be directed for tackling problems with non-binding constraints for prescribing a number of assignment.

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