Learning Geometry Obstacle

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Obstacles in Constructing Geometrical Proofs of Mathematics-Teacher-Students Based on Boero's Proving Model

Obstacles in Constructing Geometrical Proofs of Mathematics-Teacher-Students Based on Boero's Proving Model

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Abstract. This study aimed at identifying the obstacles of mathematics-teacher-students based on Boero's proving model. This study was conducted using a mix method by applying sequential explanatory strategy. The stages of the research were carried out by taking the quantitative data and revealing the qualitative data using semi-structured interviews. In the result of this study, it was found that most of mathematics-teacher-students had difficulties in constructing geometrical proofs of each Boero's proving model. Even in the phase of writing formal proof, there were only 6.67% of students could write fully in the cases of indirect proving. There were 13.33% of students in the cases of direct proving. This study concluded several obstacles which students faced in constructing the geometrical proofs formally in each phase of Boero' proving model. The obstacles included: the difficulty in making a diagrammatic sketch of conjecture which was completely made with the correct geometrical notation; the difficulty in knowing of cause-effect of geometrical problems to be proved, if it involved some conditional sentences; inability to write a conjecture made in the form of geometrical symbols, formulas and axiomatic deduction; the difficulty in selecting a valid statement of the conjecture made and the difficulty in writing formal proof.

1. Introduction

When we talk about mathematics, it is implied in mind that it is a science which requires us to use the logic of mathematical rules in the form of postulates and theorems to analyze and solve all the problems of algebra and geometry to form a concept or new mathematical knowledge. Since mathematics is the science of logic on the form, composition, scale, and concepts related to each other which is divided into three areas, namely algebra, analysis and geometry [1], it requires a treatment so that the construction of a concept can be understood by students.

Studying mathematics is to study the branches of mathematics which are one of them is the geometry. Everything in this universe is a geometrical case. Therefore, mathematics through the geometry is to be learned about the concepts embodied in the objects which exist in nature through the concepts of geometry. Geometry is very important to be developed in learning process of mathematics in classroom. The objectives of learning geometry are as follows [2]:

1. Developing spatial ability, geometrical intuition and the ability to visualize geometrical forms.
2. Increasing knowledge of geometry in two and three dimensions.
3. Developing knowledge, understanding and ability to use geometrical properties and theorems.
4. Encouraging the development, the use of conjecture, the deductive reasoning and geometrical proofs.
5. Developing the ability to apply geometry through modeling and problem solving in a real-world context.
6. Generating positive attitudes towards mathematics.
7. Developing awareness of the historical and cultural heritage of geometry in the society, and the application of contemporary geometry.

As it has been mentioned above about the learning objectives of geometry, one of crucial elements in learning geometry is the development of deductive reasoning and geometrical proving. Reasoning and proving are fundamental parts of mathematics and deeply rooted in the different areas of mathematics. Both are to be understood by those who are studying geometry. The components of geometry in curriculum of mathematics should not only put the process of building a spatial visualization and the ability of mathematical understanding, but also put their capacity in developing deductive reasoning and proving [3–8]. Even at a basic level (for ages 14-16 years), it is important to teach the students about geometrical proving to distinguish between practical demonstration and proving showing the steps of deduction toward geometrical problems [7].

However, verifying the geometrical proving lately becomes an obstacle which the students seem hardly developed. They have a difficulty in analyzing geometrical characteristics which are considered in the form of theorems. Meanwhile, in various studies, the ability of students about to verify the geometrical proving has not yet reached as what it is expected and students still have limitations in constructing the proof formally [9–11]. The limitations of students in mathematical proving can be seen in the mistakes which typically occur in constructing the proof, namely: 1) that students only work on specific cases with inductive rather than using deductive argument, 2) the argument which is still considered the results to be proven, and 3) that students determine the argument which contains invalid assumptions [12]. In addition, a study also revealed some of the difficulties which mathematics-teaching-students have in constructing the proof, namely: 1) the difficulty in understanding and expressing definition; 2) the limitations of intuition related to the concept to be proven; 3) the lack of concepts of students to compile the proofs; 4) the inability to make their own examples to clarify the proofs; 5) the ignorance in understanding the benefits of definition as an ingredient to write a proof; 6) the inability to use the language and symbols or mathematical notation; 7) the ignorance to start verifying the proof [13].

The findings about students' difficulties in constructing proofs were also explained by Selden and Selden [14]. He concluded from his research that the students in building the proof was still very disappointing that only 40% of students showed proof of deductive reasoning to the usual result, and only 10% of student showed the proof for unusual result. Nevertheless, most of the students (84% in geometry, 62% in algebra) have realized that whenever the statement has been proved, there is no further work needed to examine whether it is applied to a variety of specific examples. Meanwhile, the result of a research which was conducted by Senk in Sumarmo [15], unveiled from 1,520 of high school students, there were only 30% of students mastered 75% in the written proof in Euclidian geometry, and only 3% of those obtaining the ideal score. And the result of research conducted by Martin & Harel (in Hinze and Reiss) [16] revealed 101 college students, only 10% of students rejected all inductive arguments, 80% of them gave a positive evaluation in using an inductive argument. Deductive argument is generally better than the inductive argument.

Geometrical proving is very important in learning geometry because of the roles of proof in mathematics, namely: 1) to verify that a statement is true, 2) to explain why a statement can be said to be true, 3) to establish communication of mathematics, 4) to find or make new math and 5) to make a systematic statement in axiomatic systematization [11].

Jones & Rodd [7] gave detail that a proof in geometry is a valid argument that establishes the truth of the statement. Most of the proof in the geometry relies on logic. It means that they are based on a series of statements which are considered true. Deductive reasoning uses the laws of logic to link all the correct statements to a correct conclusion. A definition is a true statement to use in geometric proof.

In relation to the ability to verify proof in general mathematics in which there is the ability of geometrical proof, Sumarmo [15] divided it into two, as follows: the ability to read the proofs and the ability to construct the proof. The ability to construct the proof is to formulate a mathematical statement of proof based on definitions, principles and theorems, and write it in the form of proof completely.

To construct geometrical proving, Ridgway [17] stated there are five steps, namely:

1. Identifying what information given and what to be proven. It is the easiest step to determine the conjecture of a causal link that contains the statement "If ..... then .....". The word "if" shows what information is given. Meanwhile, the word "then" indicates the part which must be proven. If the conjecture is not suitable with what is given, we can restate in the form of "if .... then ...".
2. Making a diagram of information given to clarify what to be proved.
3. Presenting back the information on the diagram created.
4. Making a plan of verifying proof by reasoning.
5. Using the plan of verifying proof created to further writing a proof.

Boero (in Reis and Reinkl) [18] explained a model of the verifying a proof consists of six phases; determine the conjecture (conjecturing), formulate a statement (formulating statement), exploration (exploring), selecting and combines coherent argument (the selection and combination of coherent arguments), test the result (testing the result), write a formal proof (writing a formal proof). In the first stage, it is to determine the allegation or conjecture to explore issues which aims to establish and identify the arguments to support the proof. The second phase is to formulate a statement which aims to provide the exact formula of the allegations made, which is the basic for further process. The third phase is to explore allegations and identify the correct arguments to be validated. The fourth phase is to select and combine the arguments determined which are coherent in deductive chain. The fifth phase is to test the result of the proof whether or not suitable with the rules of mathematics which can then be written into formal proof in the sixth phase.

In Indonesia, geometry in curriculum of college is divided into several subjects in each semester. At the beginning of the semester (second semester), student learn geometry through a flat geometry subject (basic geometry). The subject of flat geometry contains of Euclidean geometry material involving axiomatic systems and geometrical proving. The ability to construct geometrical proving in this subject needs to be developed.

Considered by the importance of geometrical proving for mathematics-teaching-students, it is important to conduct a study to identify the obstacles faced by mathematics-teaching-students in Indonesia. It becomes a question whether the characteristics of the obstacles which have been revealed would be the same as the characteristics of the mathematics-teaching-students in Indonesia or there were other findings. Therefore, it is important for assessing the ability of constructing a geometric proof of mathematics-teaching-students in order to obtain information about obstacles.

With reference to the steps of verifying geometrical proving disclosed by Ridgway and the Boero's proving model, to identify obstacles of mathematics-teaching-students in constructing proofs based on Boero's proving model includes the following: knowing information given and what is to be proved; determining the cause-effect of the problem to be proven; describing a diagrammatic sketch completely with geometrical symbols and notations; making a statement based on known information and diagrammatic sketch made; writing a conjecture that has been made in the form of geometric symbols, formulas and deductive axioms; exploring valid and invalid statements of conjecture which have been made; choose a valid statement of conjectures made; creating a link between the valid statements using deductive axiom rules; testing beforehand.

2. Method

This study was conducted using a mixture (mix method) by applying explanatory sequential strategy [19]. In the first stage of quantitative data collection, the samples consisted of 30 mathematics-teaching-students at a flat geometry subject at the University of Muhammadiyah Prof. DR HAMKA the second semester of the 2015/2016 academic year. Each student was given two pieces of geometrical proving test with one test on indirect proof (contradiction) and one test associated with the direct proof. The following tests of geometrical proving were submitted:

Geometrical proving test 1 (indirect)
Prove if the triangle of ABC was determined $\angle A = \angle B$, *then* AC=BC.

Geometrical proving test 2 (direct)
Prove if the triangle of ABC was determined the center point P at side of AC, then line // AB through point P will cut BC in point Q exactly at the center.

Furthermore, students' answers were given 1 of score for a correct answer and 0 for an incorrect answer on each indicator. Students' answers were analyzed with descriptive statistics referring to the indicator of the ability to construct geometrical proofs as can be seen in Table 1.

Table 1. Indicators of Constructing Ability Geometrical Proof (Adapted from Boero)

No.	Phase	Indicators
1	*Conjecturing*	Knowing the information given and what is to be proved
		Knowing the cause-effect of the problem to be proved.
		Describing diagrammatic sketch with geometrical symbols and notation completely
		Making statements based on provided information and diagrammatic sketch made
2	*Formulating Statement*	Writing down conjectures made in the form of geometrical symbols, formulas and axiomatic deduction
3	*Exploration*	Exploring the valid and invalid statements of conjecture made
4	*Selection and combination of Coherent Arguments with Deductive Chain*	Selecting valid statements of conjectures made
		Creating a link among valid statements using the rules of axiomatic deduction
5	*Testing Result*	Testing the first geometrical proof before writing it down formally
6	*Writing Formal Proof*	Using the rules of method of proving (direct and indirect) correctly
		Writing down formal geometrical proof completely

The next stage was to explore the data qualitatively by conducting interviews to four students. The qualitative data were taken from semi-structured interviews in order to obtain the obstacles in constructing the geometrical proofs. The questions were asked related to indicators of the ability to construct geometrical proof. Furthermore, the data from interviews were analyzed to determine the obstacles on the ability to construct geometrical proofs of mathematics-teaching-students.

3. Result and Discussion

The result of quantitative data was analyzed with descriptive statistics. After that, it was conducted an interview to several students based on the answers written to get the qualitative data about the ability to construct geometrical proof.

3.1. Phase of Conjecturing

This phase involved four indicators, namely: knowing the information given and what is to be proved; knowing the cause-effect of the problem to be proved; Describing diagrammatic sketch with geometrical symbols and notation completely; making statements based on provided information and diagrammatic sketch made. Table 2 is the result of the students' answers at each geometrical proof test:

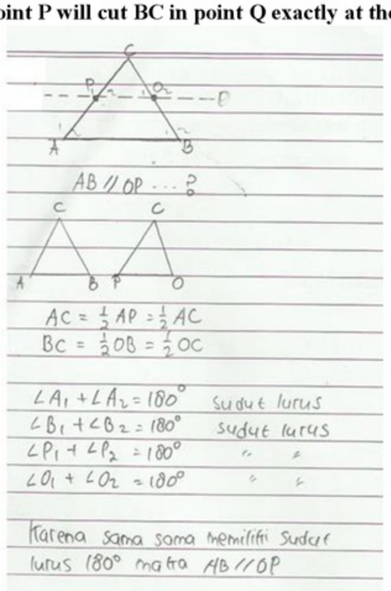
Table 2. The Percentage of Students' Correct Answers in Phase of Conjecturing

Phase	Indicators	Geometrical Proof Test 1		Geometrical Proof Test 2	
		Total	Percentage (%)	Total	Percentage (%)
Conjecturing	Knowing the information given and what is to be proved	28	80.00	20	57.14
	Knowing the cause-effect of the problem to be proved	26	74.29	19	54.29
	Describing diagrammatic sketch with geometrical symbols and notation completely	12	34.29	15	50.00
	Making statements based on provided information and diagrammatic sketch made	11	31.43	12	34.29

Table 2 above shows that 80% and 57. There were 14% of students answered correctly for the indicator of knowing the information given and what is to be proved. Overall, students could determine what information is known and could understand the problem to be proven. On indicators of knowing the cause-effect of the problem to be proved, Geometrical Proof Test 1 by 74, 29% of students answered correctly. That was caused by Geometrical Proof Test 1 which only involved two simple statements with a triangle of ABC with a cause and a side of AC = BC as an effect to be proved. However, in the Geometrical Proof Test 2, there were only 54.29% of the students knew the cause-effect of the problem to be proved. That was because the Geometrical Proof Test 2 was more complex.

Geometrical Proof Test 2 (direct)

Prove if the triangle of ABC was determined the center point P at side of AC, then line // AB through point P will cut BC in point Q exactly at the center.



Prove if *AB//OP?*

$$AC = \frac{1}{2} AP = \frac{1}{2} AC$$

$$BC = \frac{1}{2} OB = \frac{1}{2} OC$$

$$\angle A_1 + \angle A_2 = 180^0 \ (\textit{straight})$$

$$\angle B_1 + \angle B_2 = 180^0 \ (\textit{straight})$$

$$\angle P_1 + \angle P_2 = 180^0 \quad ,, \quad ,,$$

$$\angle O_1 + \angle O_2 = 180^0 \quad ,, \quad ,,$$

Because both have a straight angle, in 1800 then AB // OP

Figure 1. Endah's Answer

From Figure 1, it can be seen that Endah had a wrong conjecture. She considered the problem to be proved is AB // OP and Endah did not write down the information known of the problem to be proved. To obtain the data deeper from Endah, the researcher conducted an interview. Here is the result of an interview with Endah.

Researcher : *From the answer that you have written, why did not you write down the information known?*
Endah : *I do not quite understand about the problem no 2 Sir.*
Researcher : *Which part, you did not understand?*
Endah : *Actually if there is a triangle of ABC with P in center point I understand Sir. But, for the next sentence "// then the line AB through the point P will cut BC at point O is right in the middle point anyway" I do not understand. I am confused which to be proven.*
Researcher : *It can be seen that you wrote the problem to be proved was AB // OP and you also wrote that line // AB cuts BC in point O. Meanwhile, the information was clear that the line // AB cuts BC at the point Q, are you sure with your allegations?*
Endah : *Actually I am not sure Sir. I am confused. As long as I knew, there was a parallel line to AB through point P, so that I was wrong in giving the symbols to writing it. Yes Sir, I symbolized it with point O.*
Researcher : *According to the dictionary, what is the meaning of the phrase "line //AB through the point P will cut BC at the point Q right in the middle point"?*
Endah : *The line through the point P and parallel to AB and cut BC at the point Q, so that I proved the line PQ // AB.*

Endah seemed confused about the meaning of the phrase "then the line //AB through the point P will cut BC at the point Q right in the middle point". Therefore, Endah merely perceived the problem as "If the ΔABC was determined the center point P on the side of AC, then the line // AB is through the point P ". Thus, Endah assumed that the center point Q of BC as a result of BQ // AB. Therefore, the problem proven is PQ // AB. In other word, Endah did not understand about the meaning of a sentence or phrase on the problem to be proved. In

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addition, Endah also failed to specify a conditional statement in the form of cause-effect of the problem. Therefore, she had a difficulty in identifying the information known and the problem to be proved.

On the indicator of describing diagrammatic sketch with geometrical symbols and notation completely, most students faced a difficulty that there were only 34.29% and 50% of students answered correctly. The students had a difficulty in describing diagrammatic sketch which was caused them unable to determine the conjecture solution of proof. The difficulty which the students had was that they described a wrong diagram which was not suitable with what was to be proved and wrong in geometrical notation. In addition, students were also unable to make statements about a theorem which would be used to solve the problem. For the last indicator which was making a statement using geometrical axiomatic system related to the solution of proof, there were 31.43% for geometrical proof test 1 and 34.29% for geometrical proof test 2 answered correctly.

Here is presented the results of the answers to student named Dewi in Figure 2.

Geometrical Proof Test 1 (indirect)

Prove if the triangle of ABC was determined $\angle A = \angle B$, *then* AC=BC.

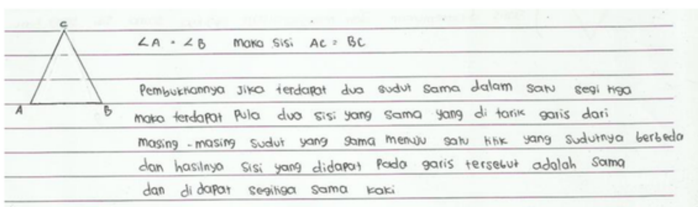

Figure 2. Dewi's Answer

$\angle A = \angle B$, then AC=BC
The proof is if there are two equal angles in a triangle, then there are also two sides of the same drawn a line from each corner toward the same point with a different angle. The result obtained on the side of the line is the same and can be concluded as an isosceles triangle.

To explore the answer of Dewi, the writer interviewed her. Here is the result of an interview with her.

Researcher : *What kind of conjecture do you think to prove this problem?*
Dewi : *If in a triangle, the two angles are equal, then two sides are also the same anyway. The sides are pulled from a common point, namely the point C, to each corner. Therefore, it forms an isosceles triangle of ABC.*
Researcher : *Please try to give any reasons for your answer.*
Dewi : *In the geometrical proof test 1, it is clear that if the two corners of a triangle are equal, the two sides are the same because it is the nature of a triangle. There is no need to prove it.*

It can be seen that Dewi considered that the statement did not need to prove. Dewi assumed that triangle in the statement was an isosceles triangle with its nature of the same feet or two sides. Therefore, according to Dewi it needed obviously no prove again.

3.2. Phase of Formulating Statement

Table 3 shows the result of students' answers of phase of formulating statement, as follows:

Table 3. The Percentage of Students' Correct Answers in Phase of Formulating Statement

Phase	Indicators	Geometrical Proof Test 1		Geometrical Proof Test 2	
		Total	Percentage (%)	Total	Percentage (%)
Formulating Statement	Writing down	10	28.57	11	31.43

	conjectures made in the form of geometrical symbols, formulas and axiomatic deduction				

Table 3 presents that there were only 28.57% of students answered correctly in Geometrical Proof Test 1 and 31.43% in Geometrical Proof Test 2. The difficulty in describing diagrammatic sketch was the first cause which in writing the conjecture made using geometrical symbols, formulas, and axiomatic deduction. The incorrect diagrammatic sketch made students unable to write a proof.

3.3. Phase of Exploration

Table 4 shows the result of students' answers of phase of exploration, as follows:

Table 4. The Percentage of Students' Correct Answers in Phase of Exploration

Phase	Indicators	Geometrical Proof Test 1		Geometrical Proof Test 2	
		Total	Percentage (%)	Total	Percentage (%)
Exploring	Exploring the valid and invalid statements of conjecture made	10	28.57	10	28.57

In this phase, it can be seen that almost all the students could not identify statements which are valid or invalid from conjecture which was made. It can be assumed by that there were only 28.57% of students answered correctly in each geometrical proof test given. The result was in line with the second phase in which can be indicated as if the second phase could not be passed, the further phases would be the same.

3.4. Phase of Selection and Combination of Coherent Arguments with Deductive Chain

Here is the result of students' answers of phase o selection and combination of coherent arguments with deductive chain, as follows:

Table 5. The Percentage of Students' Correct Answers in Phase of Selection and Combination of Coherent Arguments with Deductive Chain

Phase	Indicators	Geometrical Proof Test 1		Geometrical Proof Test 2	
		Total	Percentage (%)	Total	Percentage (%)
Selection and combination of Coherent Arguments with Deductive Chain	Selecting valid statements of conjectures made	11	31.43	10	28.57
	Creating a link among valid statements using the rules of axiomatic deduction	9	25.71	9	25.71

Table 5 presents only 31.43% of students answered correctly in the Geometrical Proof Test 1 and 28.57% in the Geometrical Proof Test 2. It indicated that the students had a difficulty in selecting a valid statement of conjectures made. The problem appeared because of the inability of students to make conjectures in the form of axiomatic deduction and the lack of students' ability in sketching graphs of conjectures made. Thus, the students also had a difficulty in making a link among the valid statements by using rules of axiomatic deduction. There were only 25.71% of students answered correctly in each geometrical proof test. The mistake which students

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made in this phase was in making a diagram of the conjecture which was made. Such mistakes are made by Ratna. Here are the answers that have been written Ratna.

Geometrical Proof Test 2 (direct)

Prove if the triangle of ABC was determined the center point P at side of AC, then line // AB through point P will cut BC at point Q exactly at the center.

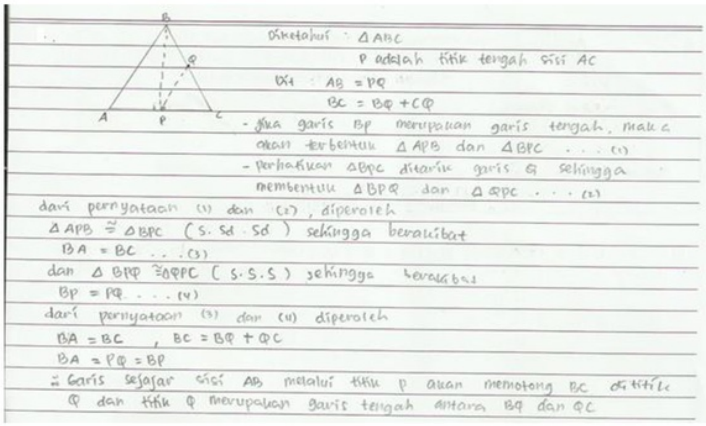

Figure 3. Ratna's Answer

Known: ΔABC
P is the center point of AC
To be proved: AB=PQ
BC=BQ+CQ
If line of BP is the center point, then it will form ΔAPB *and* ΔBPC *. . . (1)*
Notice ΔBPC *the line of Q was drawn so it will form* ΔBPC *and* ΔBPC *....(2)*
From statement (1) and (2),
obtained $\Delta APB \cong \Delta BPC$ *(S.Sd.S) so it will causes BA=BC... . (3)*
and (S.S.S) so $\Delta BPQ \cong \Delta QPC$ *causes BP=PQ . . .(4)*
from statement (3) and (4) obtained
BA=BC , BC = BQ+QC
BA=PQ=BP
$\therefore$ **The line //** ***AB*** **through point** p **will cut** ***BC*** **at point** ***Q*** **and point** ***Q*** **is the center line between** ***BQ*** **and** ***BC***

From Figure 3, it can be seen that Ratna was wrong in describing the diagrammatic sketch. The conjecture of problems to be proven was also incorrect. As the result, she was unable to make a valid statement and use the geometrical theorem to justify the statement which was made. Thus, the statement linked no connection with each other at all, even somewhat it seemed forced and made up. To obtain the further data about Ratna's answer, the writer conducted an interview. Here is how the interview with Ratna conducted.

Researcher	*: Are you quite sure about the proof you wrote?*
Ratna	*: Actually I am not sure Sir. I just wrote what I knew it then I applied what I know to the question.*
Researcher	*: Are you sure that you answer is correct?*
Ratna	*: Not quite sure Sir.*
Researcher	*: do you really think it is correct the statement you made that "If the line of BP is the center line, it will form* ΔAPB *and* ΔBPC*, (1) and* ΔBPC *the was pulled a line of Q so it will form* ΔBPC *and* ΔBPC *(2), then* $\Delta APB \cong \Delta BPC$ *(S.Sd.S) it will give result BA = BC.*

Ratna : *True Sir. It was already qualified that two mutually congruent triangles are S.Sd.S.*
Researcher : *Okay let's see, where are the two sides which are the same, and where are the angles which are the same?*
Ratna : *AB=BC, AP=CP and* $\angle APB = \angle CPB$..

From the interview with Ratna, she was actually not sure with her conjecture. However, Ratna kept pushing with statements according to what she knew. She did not know that the statements she made were valid or not. Ratna tried to relate all the statements which she made although there was no connection to link invalid statements to valid ones. In addition, Ratna also kept trying to relate all which she did not master with the problem to be proved. It can be seen that Ratna tried to pull line of BP so that AP = CP and AB = BC. In this case, the problem to be proved by her was not an isosceles triangle but it was a scalene triangle.

3.5. Phase of Testing Result
Here is the result of students' answers of phase of testing result, as follows:

Table 6. The Percentage of Students' Correct Answers in Phase of Testing Result

Phase	**Indicators**	**Geometrical Proof Test 1**		**Geometrical Proof Test 2**	
		Total	**Percentage (%)**	**Tot al**	**Percentag e (%)**
Testing Result	Testing the geometrical proof before writing it down formally	7	20.00	9	25.00

Table 6 shows that there were only 20.00% of students tested geometrical proof test in geometrical proof test 1 and there were 25.00% in geometrical proof test 2. Most of the students wrote the formal proof without tested it first after they got confident with the conjecture they made was right.

3.6. Phase of Writing Formal Proof
Here is the result of students' answers of phase of writing formal proof, as follows:

Table 7. The Percentage of Students' Correct Answers in phase of Writing Formal Proof

Phase	**Indicators**	**Geometrical Proof Test 1**		**Geometrical Proof Test 2**	
		Total	**Percent age (%)**	**Total**	**Percent age (%)**
Writing Formal Proof	Using the rules of method of proof (direct and indirect) correctly	7	20.00	9	25.71
	Writing down formal geometrical proof completely	2	6.67	4	13.33

Table 7 presents that there were 20.00% of students knew the proof used was indirect proof (contradictory proof), but there were only 6.67% of students wrote the formal proof completely. Meanwhile, in the geometrical proof test 2, there were only 25.71% of students knew the proof used is direct proof and there were 13.33% of students wrote in formal proof completely. In the first problem, students thought that all the problems of geometrical proof could only be solved by direct proof. Nonetheless, most students seemed did not understand to verify the geometrical proof using direct proof yet. As shown below Risky's answer which uses contradictory proof to solve the problem 1, but the Risky's answer was yet incomplete and found a few flaws. Here is the answer of Risky.

Geometrical Proof Test 1 (indirect)

Prove if the triangle of ABC is determined $\angle A = \angle B$**, then side of** AC=BC.

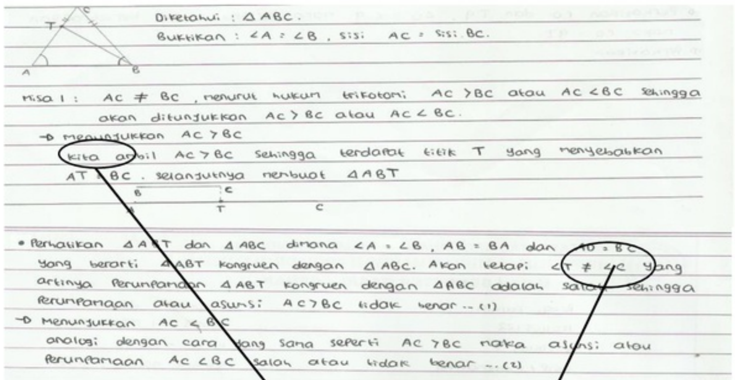

Figure 4. Risky's Answer

Known: ΔABC
Shown: $\angle A = \angle B$ *, side of AC = BC*
For example: $AC \neq BC$ *according to the rules of trichotomy AC>BC or AC<BC so it will be indicated as AC>BC or AC < BC.*

- *Indicated as AC > BC*

We take AC > BC so it emerges point T which causes AT=BC, and makes ΔABC *.*

- *Notice* ΔABT *and* ΔABC *where* $\angle A = \angle B$ *, AB=BA and AD=BC which means* ΔABT *is congruent with* ΔABC *. But* $\angle T \neq \angle C$ *means that the parable of* ΔABT *is congruent with* ΔABC *is wrong so the assumption of AC>BC is not true.....(1)*
- *Indicated as AC<BC*
  The analogy with the same way as AC> BC then the assumption of AC <BC is not true....... (2)

The writer tried interviewing Risky based on answer shown in Figure 4, focusing on the incompleteness of Risky's answer to the conclusion of the proof. Here is the interview with Risky.

Researcher : What method of proof did you use?
Risky : Contradictory proof sir.
Researcher : Was the proof which you wrote complete?
Risky : I think it is complete sir.
Researcher : I could not find the final conclusion from the proof you wrote. Can you explain?
Risky : Oh, yes sir, actually statements (1) and (2) it means the assumption of $AC \neq BC$ *was wrong. What is correct is AC = BC, which means it was proved, sir.*
Researcher : I also found an error in the statement you previously wrote that AT = BC, but on the next statement AD = BC. What did it mean?
Risky : That's correct sir AT = BC, I was wrong to write the statement below. I am sorry I was insecure sir.
Researcher : Did you first check your answer before writing it down formally?
Risky : No sir, I wrote my idea and the proof sir without checking my answer first.

From the interview conducted to Risky, it can be seen that he was careless in writing the formal a proof. It can be obviously seen from the writing from the mistakes in each statement. In addition, Risky considered that the proof which he wrote was complete. Meanwhile, it was important in writing a formal proof to write it to the conclusion that the problem was already verified. Risky felt confident of the answer. Risky's carelessness was that he did not re-check the proof which he wrote. He was quite sure with his answer so that he did not have to re-check.

This study tried to identify obstacles in constructing geometrical proof based on Boero's proving model. From the result of data analysis both qualitative and quantitative, it showed that phase of conjecturing determined further subsequent phases. It can be seen that most of students had a difficulty in determining the correct conjecture with the problem to be proved. In phase conjecturing, understanding the information and what to be proved was extremely crucial. Herbst [20] stated that an authentic aspect of mathematics is a dynamic relationship between the information known and what to be proved as basic information to determine the correct conjecture. In knowing the information known, the students did not have any problems. However, to determine the problems to be proved, the student had a little problem. Such obstacle happened if students were about to face geometrical proof problems which were more complex involving some conditional sentences. It means that students had a difficulty in knowing the cause-effect of geometrical problems to be proved if it involved some conditional sentences. Therefore, it became an important concern in constructing geometrical proof of identifying information to determine the conjecture in a statement of cause-effect of geometrical problems "if .... then ...." in constructing the geometrical proof [17].

Describing diagrammatic sketch with geometrical notations was an important part in the phase of conjecturing. Allegations which students made would be a valid argument if it was started from the correct diagrammatic construction. Ridgway [17] explained that making a diagram of information provided could clarify what to be proved. However, from the data analysis, students had a difficulty in making a diagrammatic sketch of the conjecture made with the correct geometrical notations. It would affect to the result of students having in making statements using axiomatic geometrical system. If the diagrammatic sketch was made incorrectly, it would affect the ideas of proof. In line with the findings, the students who made the diagrammatic sketch incorrectly made the incorrect ideas in the statements of proof with geometrical axiom system. The findings were in line with by Dvora & Dreyfus [21] that the making of the definition of a concept of geometry should include standard diagrams. The step should be done or else it could cause difficulties in interpreting the diagrams made.

In the phase of formulating the statement, the problem which arose was an error in writing the conjectures made in the form of geometrical symbols, formulas and axiomatic deduction caused by incorrect diagrammatic sketch. Phase of formulating statement would affect the next further phase in exploring the valid or invalid statements from the conjecture made. Overall, students went through this phase but the effectiveness in analyzing and exploring the conjecture made were still lacking. The process of exploration was done only based on geometrical knowledge of students which they mastered. The lack of experience and knowledge of geometrical proof with axiomatic system was a major cause of mathematics-teaching-students. Therefore, sometimes statements which students made did not have a valid reason to use axiomatic system.

The next phase was the selection and combination of coherent arguments with deductive chain. In this phase, students had a difficulty in choosing a valid statement of conjecture which was made. The problem appeared from students' inability to make conjectures in the form of axiomatic deduction and the lack of students in sketching graphs of conjectures which was made. Thus, the students had a difficulty in making a link among the valid statements.

The phase of testing result was forgotten by most students. Most students quickly wrote conjectures made and then the statements which they thought it was correct. Though, it would affect the next last phase of writing formal proof which was the ultimate goal of constructing geometrical proof. In writing formal proof phase, it could not be separated from the proper use of the proof method. Accuracy in writing a formal proof was an obstacle which students had. It is because the absence of a process of re-checking the formal proof that was written. In addition, in the result of data analysis, the most students considered that the problems of geometry could only be proven by direct proof only so when they faced the problem of indirect proof, students had a difficulty in starting to write a proof.

From the data analysis and discussion, it can be concluded that some of the obstacles faced by mathematics-teaching-students in constructing the geometrical proof include:

1. Difficulty in making a diagrammatic sketch of a conjecture made completely with the correct geometrical notation.
2. Difficulty in knowing the cause-effect of geometrical problems to be proved, if it involved some conditional sentences.
3. Inability to write a conjecture made in the form of geometrical symbols, formulas and axiomatic deduction.
4. Lack of experience and knowledge of the geometrical knowledge with axiomatic system.
5. Difficulty in choosing a valid statement of conjecture made.
6. Assumption of geometrical problems can only be proven by direct proof so when they faced the problem of indirect proof, students had a difficulty in starting to write a proof.
7. Difficulty in writing formal a proof.

4. Conclusion

Mathematical proofs is an activity that needs to be developed in the process of learning mathematics. By proving, students can develop the thinking to the understanding of a mathematical concept. However, proving geometry has many obstacles in the learning process. Students experienced some obstacles in constructing a formal proof of the geometry in each phase of verification by the Boere. First, the difficulty in making a sketch diagram of conjecture complete with the correct geometry notation. Second, difficulty in knowing the causal relationship of geometrical problems to be proved if involving some conditional sentences. Third, lack of ability of the students to write a conjecture that has been made in the form of geometric symbols, formulas, and deductive axiomatic geometry. Fourth, difficulty in choosing a valid statement of the conjecture that has been made and the difficulties in writing a formal proof.

5. Recommendations

From the result of the identification of students' obstacles in constructing the geometrical proof which had been disclosed above, the researcher would like to offer some recommendations in constructing the geometrical proof, namely:

1. Understand the problem to be proved by writing keywords of the problem to be proved it, then think of the characteristics of the key words.
2. Separate some things of the problem as a conclusion.
3. Make a geometrical sketch to be proven completely with geometrical symbols related to the problem to be proved.
4. Analyze the sketch which was made, and then relate all provided information on the sketch made to some provided theorems.
5. Make a conjecture which leads to proving.
6. Select the method of proving which is appropriate to the problem to be proved (select the more effective one).
7. Relate some necessary theorems to conjecture which was made.
8. Write down the proof systematically and do not forget every statement must provide the reason.
9. Review the steps of verifying the proof. If some errors are found, fix them by relating to the concepts. If it is flawless, then the steps of verifying the proof are already acceptable.
10. Perform other way of verifying proof of instead on the one which was used.

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