IndoMath
Indonesia Mathematics Education
ISSN 2614-5103 (Print)
2614-5111 (online)
https://indomath.org/index.php/indomath
Vol 7, No. 1, February 2024, pp. 20-34

Discovery Learning In Geometry Class: The Implementation of Learning to Built Pre-Service Teachers' Conceptual Understanding of Geometry Based On Sociomathematical Norms

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ABSTRACT

It is necessary to emphasize the ability to conceptualize the understanding of geometry (CUG) for each student to support the formation of mathematical competence. In forming, it is essential to carry out a lesson by prioritizing interaction between pre-service teachers and students in conveying ideas known as sociomathematical norms. The construction of CUG is significant for learning by prioritizing interaction between students in conveying ideas known as sociomathematical norms. The study aims to analyze the implementation of the discovery learning model (DLM) in developing achievements of CUG based on aspects of sociomathematical norms in learning plane geometry. This study used a quasi-experimental method by following a nonequivalent control group design. The sample involved in this research was 70 mathematics education study program students. This study concludes: 1) Achievement of students' CUG by implementing DLM is better than that of students' CUG by applying the conventional model (CM). 2) there are significant differences between the achievement of pre-service teachers' student CUG using DLM and CM based on the sociomathematical norm level; 3) there is an interaction effect between DLM and sociomathematical norm factors on achievement of students' CUG; 4) the contribution of DLM implementation to student CUG achievement includes: (a) student worksheets with DLM syntax make it easier for students to deepen geometry material; (b) a discussion process that gives rise to students' freedom of expression; (c) the diversity of alternative problem solving strengthens students in determining the best choice and analysis of problem solving; (d) classical presentation activities motivate students to understand the concepts in each lesson. The research concludes that DLM is effectively implemented in the lecture process, especially in geometry lectures. DLM is effective for forming students' ability to understand geometric concepts by considering the aspects of socio-mathematical norms. Students with high and low sociomathematical norms can adequately participate in learning with DLM to achieve pre-service teachers' students' understanding of geometric concepts.

Keywords: Conceptual Understanding Geometry; Discovery Learning; Sociomathematical Norm

ABSTRAK

Perlu ditekankannya kemampuan pemahaman konseptual geometri (conceptual understanding of geometry yang disingkat CUG) pada setiap mahasiswa untuk menunjang pembentukan kompetensi matematika. Dalam pembentukannya, penting untuk melaksanakan pembelajaran dengan mengutamakan interaksi antar siswa dalam menyampaikan gagasan yang dikenal dengan norma sosiomatematika. Konstruksi CUG sangat penting untuk pembelajaran dengan mengutamakan interaksi antar siswa dalam menyampaikan gagasan yang dikenal dengan norma sosimatematika. Penelitian ini bertujuan untuk menganalisis penerapan discovery learning model (DLM) dalam mengembangkan prestasi CUG berdasarkan aspek norma sosio matematika dalam pembelajaran geometri bidang. Penelitian ini menggunakan metode eksperimen semu dengan mengikuti desain kelompok kontrol nonekuivalen. Sampel yang terlibat dalam penelitian ini adalah mahasiswa program studi pendidikan matematika sebanyak 70 orang. Penelitian ini menyimpulkan: 1) Pencapaian CUG siswa dengan penerapan DLM lebih baik dibandingkan pencapaian CUG siswa dengan penerapan conventional model (CM). 2) terdapat perbedaan yang signifikan antara prestasi belajar CUG mahasiswa yang menggunakan DLM dan CM berdasarkan tingkat norma sosio matematika; 3) terdapat pengaruh interaksi antara faktor DLM dan norma sosiomatematika terhadap prestasi belajar CUG siswa; 4) kontribusi penerapan DLM terhadap prestasi CUG siswa meliputi: (a) lembar kerja mahasiswa dengan sintaks DLM memudahkan siswa dalam memperdalam materi geometri; (b) proses diskusi yang memunculkan kebebasan berekspresi siswa; (c) keragaman alternatif pemecahan masalah memperkuat siswa dalam menentukan pilihan terbaik dan analisis pemecahan masalah; (d) kegiatan presentasi klasikal memotivasi siswa untuk memahami konsep dalam setiap pembelajaran. Penelitian menyimpulkan bahwa DLM efektif diterapkan dalam proses perkuliahan khususnya perkuliahan geometri. DLM efektif untuk membentuk kemampuan mahasiswa calon guru matematika dalam memahami konsep geometri dengan memperhatikan aspek norma sosio-matematika. Siswa dengan norma sosiomatematika tinggi dan rendah dapat mengikuti pembelajaran dengan DLM dengan baik dalam mencapai pemahaman konsep geometri mahasiswa calon guru matematika.

Kata Kunci: Pemahaman Konsep Geometri, Discovery Learning, Sociomathematical Norm

INTRODUCTION

The process of learning mathematics is determined mainly by the lecturer as a facilitator and a motivator in delivering lecture material. In addition, success in learning is also determined by the content of the material provided by a teacher who aims to improve student competence (Kyriazis et al., 2009; Wardono et al., 2020; Yurniwati & Hanum, 2017). Schoenfeld & Kilpatrick (2008) said there are five main mathematics competencies: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Even though the five are a unified whole, the dimension of conceptual understanding has an essential role as the primary foundation for someone to have good mathematical competence (Soro et al., 2018; Ozdemir et al., 2020; Veith et al., 2022).

Veith et al. (2022) formulated an understanding of mathematical concepts as knowledge that involves a thorough understanding of the basic concepts and procedures that apply to mathematical principles. Hrnjičić et al. (2022) stated that there is a difference between the ability to understand concepts and understand procedures. Knowledge of a material concept is often referred to as an understanding of the concept (Schoenfeld & Kilpatrick, 2008), whereas a procedural understanding focuses on one's steps to achieve mathematical goals (Idrus et al., 2022; Maarif et al., 2021). Understanding mathematical concepts and procedures must be mutually reinforcing to form mathematical competence (Vahlo et al., 2022; Yatim et al., 2022).

The importance of conceptual understanding of geometry (CUG) and procedural fluency urgently needs to be emphasized to every student in supporting the formation of mathematical competence (Schoenfeld & Kilpatrick, 2008). Veith et al. (2022) said a student can construct knowledge well by connecting procedures and concepts he understands. However, the less-than-optimal relationship between processes and mathematical concepts will hamper students' understanding of essential concepts in every lesson that is carried out (Maarif et al., 2021; Malatjie & Machaba, 2019). Kashefi et al.'s (2012) study reported the tension between calculation results and understanding the concepts in solving math problems. In addition, several studies have revealed that prospective teacher students do not fully understand in-depth important mathematical concepts, including geometry (Rachmiazasi Masduki et al., 2019), group concepts (Veith et al., 2022), abstract algebra (Subroto & Suryadi, 2018), and trigonometry (Nabie et al., 2018).

Learning geometry is closely related to understanding the concept of a geometric statement, which further requires proof (Maarif et al., 2019; Sommerhoff & Ufer, 2019). Understanding geometric concepts needs ideas outlined in statements with valid arguments. To develop this argument, a collaborative process between students is required to build understanding in social interaction through learning geometry (Widodo et al., 2019). Therefore, in each lesson, it is necessary to pay attention to the sociomathematical norm aspects to assist students in constructing their CUG.

Social interaction in learning mathematics, especially in the collaborative process of solving mathematical problems, has been emphasized over the last decade (Kang & Kim, 2016; Partanen & Kaasila, 2015; Widodo et al., 2019). in learning geometry, each student provides geometric ideas and exchanges ideas with one another. The discussion process offers opportunities for students to gather information by reading references online before ideas are put into writing (Sumarwati et al., 2020). Therefore, to build a CUG, it is necessary to consider socio-mathematical norms to reinforce the student learning process.

In the college curriculum, learning geometry is a compulsory subject that pre-service teachers must understand. CUG includes understanding the axioms, theorems and definitions listed in the axiomatic geometric system (Maarif et al., 2019; Malatjie & Machaba, 2019). However, several studies report that students still experience problems in applying concepts and understanding geometric issues that must be solved (Noto et al., 2019), representing images of geometric problems (Shodikin et al., 2019); prioritizing procedures over deepening geometric concepts (Malatjie & Machaba, 2019) and understanding concepts in the form of geometric proofs (Maarif et al., 2019).

Several research results found several obstacles for students in understanding geometric concepts. The research results of (Maarif et al., 2019) several obstacles in learning geometry, including students' difficulties in defining geometric concepts in images, challenges in understanding cause and effect relationships between geometric concepts and applying geometric concepts in proving theorems. Alghadari & Herman (2018) revealed that students' obstacles in understanding the idea of transformation geometry are the difficulty of understanding complex procedures involving several theorems in solving geometric problems. Furthermore, (Noto et al., 2019), with the results of their research, concluded that students' obstacles in understanding concepts include difficulty applying geometric concepts related to visualization, determining principles and understanding geometric problems. Therefore, it is necessary to provide treatment that can bridge these obstacles so that prospective teacher students can fully understand the concept of geometry.

A learning treatment for students that accommodates both elements is needed to bridge the balance between understanding the concepts and procedures for solving a mathematical problem. These characteristics can be obtained from learning centred on the activities of preservice teachers (Baroody et al., 2015), focusing on forming category and concept aspects to interpret problem-solving experiences (Stein et al., 2016), processing information on meaningful learning (Cervantes-Barraza et al., 2020), and establishing constructivism theory in constructing knowledge (Length, 2013; Maarif et al., 2020). In addition, the CUG can be well-formed if students' interaction in developing mathematical ideas goes well (Ningsih & Maarif, 2021). Thus, implementing learning requires a model with the previously mentioned characteristics.

Models that have characteristics in the exploration and elaboration process of CUG include the discovery learning model (DLM). The application of this model makes it possible to explore their ability to understand concepts with students in discovery activities from various valid references (Affandi et al., 2022). Students will actively discover their knowledge independently (Balim, 2009; Maarif, 2015), find problems, explore with data, find solutions, and draw conclusions (Shahbazi & Byun, 2022; Permatasari et al., 2018). Exploratory activities in DLM also activate students understanding of mathematical concepts and mathematical problem-solving procedures because students are required to collect some data related to the concepts being studied (Wardono et al., 2020).

Some research results found the effectiveness of DLM in increasing understanding of concepts, especially in terms of representation in the form of images, presenting concepts systematically (Muhali et al., 2021) and integral understanding facilitating problem-solving (Permatasari et al., 2018). The research results of Kamaluddin & Widjajanti (2019) concluded that the exploratory stage of mathematics material with discovery activities has excellent potential to develop students' understanding of concepts because of the many opportunities to explore mathematical concepts. Research conducted by (Suyitno et al., 2019) found that implementing DLM in geometry class can form an understanding of geometric contexts, especially in using mathematical models, diagrams, and symbols as well as comparisons between geometric concepts. Furthermore, Permatasari et al. (2018) conducted a study on implementing the discovery learning approach, which concluded a positive contribution to mathematical ability in terms of interpersonal intelligence. Several studies on the application of DLM to improve CUG that have been mentioned are rarely found, focusing on CUG from the perspective of sociomathematical norms. The research that has been done has a novelty in analyzing students' CUG in terms of the perspective of socio-mathematics norms by applying DLM.

This study makes aspects of socio-mathematics norms a categorical variable to see students' CUG in learning. This is intended to see how far socio-mathematical norms become a factor in developing students' CUG using the needed ideas. Thus, aspects of socio-mathematical norms can be used as a reference for categorizing students' CUG.

Based on the description above, we must analyze how students' CUG apply DLM to geometry lectures. The research was conducted to answer several research questions, namely: 1) is the achievement of students' CUG by applying DLM better than that of students who apply CM?; 2) is there a difference in achievement of students' CUG using DLM and students applying the CM based on the socio-mathematical norm level?; 3) is there any interaction effect between DLM and students' sociomathematical norm factors on achievement of CUG?; 4) how does the implementation of DLM contribute to the achievement of students' CUG?

METHOD

The study aims to analyze the implementation of the discovery learning model (DLM) in developing the achievement of students' CUG based on aspects of socio mathematical norms in learning plane geometry. This study used a quasi-experimental method following a nonequivalent control group design (Sztajn et al., 2013). The quasi-experimental design uses a nonequivalent control group design (Cohen et al., 2002), as shown below.

Experiment	O_1	X	O_2
Control	O_3		O_4

Participant

This research involved 70 students of the mathematics education study program in plane geometry lectures at a private university as a research sample taken from 478 populations. The sampling technique was carried out using cluster random sampling. 35 student samples were used as an experimental class, while 35 were used as a control class group. Learning treatments were given to the practical class by implementing DLM while the control class was using CM.

Procedure

Each research class was given a test to measure the achievement of students' CUG by first analyzing the sociomathematical norm aspect as a control variable. The sociomathematical norm analysis was taken from a questionnaire filled in by students first. Furthermore, the results of the socio-mathematical norm questionnaire scores were calculated by sorting the lowest score to the highest score. Then, the sociomathematical norm score was categorised by considering the standard categorization (Wardono et al., 2020) as in Table 1.

Table 1. Sociomatematical Norm Aspect Score Category

Questionnaire Score Interval	Category
$\boldsymbol{x} \geq (\boldsymbol{\mu} + \boldsymbol{\sigma})$	High
$(\boldsymbol{\mu} + \boldsymbol{\sigma}) < \boldsymbol{x} < (\boldsymbol{\mu} - \boldsymbol{\sigma})$	Medium
$\boldsymbol{x} \leq (\boldsymbol{\mu} - \boldsymbol{\sigma})$	Low

This study uses two different treatments. The experimental class gets the learning treatment by applying DLM, while the control class gets the CM. Each class of students received material 7 meetings for seven weeks with different geometry materials at each meeting. The materials studied at each meeting can be shown in Table 2.

Table 2. Distribution of Learning Materials

Meeting	Subject matter	Sub Material
1st	Line and angle	Lines and angles
2nd	Parallel line	The concept of parallel line and its consequences
3th	Triangle	The relationship between the angles in a triangle and the special lines of a triangle
4th	Triangle	Congruence of triangles
5th	Polygon	The concept of polygons and parallelograms
6th	Polygon	The concepts of Rectangle, square, rhombus and trapezoid
7th	Area	Area of a triangle and a quadrilateral

In the experimental class, each meeting applies DLM with learning steps including Prerequisite, Identifying problems, Planning and collecting data, Analyzing data, Planning solutions, Presenting, and Concluding (Abrahamson & Kapur, 2018; Kamaluddin & Widjajanti, 2019; Maarif, 2015; Siregar et al., 2020)—the learning activities are shown in Table 3.

Table 3. Stages of learning with DLM

Stages	Activity
Prerequisite	Students are divided into several groups, with several groups of 4-5 students. Before carrying out learning activities, students have prerequisite knowledge of geometry material obtained in high school. Then, each group is given a student worksheet containing the geometric problems to be explored
Identifying problems	In groups, students identify geometric problems that are presented on student worksheets
Planning and collecting data	Students explore the problems presented on student worksheets by collecting data such as information known in the issues raised to form concepts by way of discussion between groups
Analyzing data	Students discuss and analyze alternative solutions to the problems presented
Planning solution	Students look for the best solution to the problems presented
Presenting	One student representing each group presented their work for further comments from other groups
Concluding	Classically, lecturers and students alike conclude concepts from the material that has been studied

Collecting Data

Data on the achievement of students' CUG was collected using a geometric concept understanding ability test. In addition, to determine the category, data is taken from the socio-mathematical norm questionnaire. The instruments used in this study were in the form of a test instrument for understanding geometric concepts and a sociometric norm questionnaire. The test for CUG is structured based on indicators of applying geometric formulas in simple calculations, doing geometric calculations algorithmically and associating one geometric concept with another and realizing the process being carried out (Malatjie & Machaba, 2019; Veith et al., 2022). The test for understanding geometric concepts consists of 4 test items. As for the sociomathematical norm questionnaire instruments, they include indicators of attitudes towards mathematical experience, mathematical explanations, mathematical differences, mathematical communication, mathematical effectiveness, and mathematical insight (Ningsih & Maarif, 2021; Yackel & Cobb, 1996; Zembat et al., 2015). The sociomathematical norm questionnaire instrument consists of 30 items. Before the two instruments were used, the validity and reliability of the instruments were first tested. The results of testing the validity of the pre-service student CUG instrument and the socio-mathematical norms questionnaire are shown in Table 4.

Tabel 4. Validity Test Results

Variabel	Item	Outer Loading	Criteria	Simpulan
CUG	1	0.810	>0.700	Valid
	2	0.805	>0.700	Valid
	3	0.812	>0.700	Valid
	4	0.747	>0.700	Valid
Sociomathematical Norm	1	0.766	>0.700	Valid
	2	0.732	>0.700	Valid
	3	0.821	>0.700	Valid
	4	0.745	>0.700	Valid
	5	0.881	>0.700	Valid
	6	0.788	>0.700	Valid

Variabel	Item	Outer Loading	Criteria	Simpulan
	7	0.172	>0.700	Drop
	8	0.792	>0.700	Valid
	9	0.782	>0.700	Valid
	10	0.206	>0.700	Drop
	11	0.734	>0.700	Valid
	12	0.716	>0.700	Valid
	13	0.812	>0.700	Valid
	14	0.850	>0.700	Valid
	15	0.102	>0.700	Drop
	16	0.715	>0.700	Valid
	17	0.794	>0.700	Valid
	18	0.776	>0.700	Valid
	19	0.205	>0.700	Drop
	20	0.786	>0.700	Valid
	21	0.782	>0.700	Valid
	22	0.882	>0.700	Valid
	23	0.124	>0.700	Drop
	24	0.823	>0.700	Valid
	25	0.874	>0.700	Valid
	26	0.027	>0.700	Drop
	27	0.804	>0.700	Valid
	28	0.789	>0.700	Valid
	29	0.807	>0.700	Valid
	30	0.802	>0.700	Valid

Table 4 shows that all pre-service teachers' students' CUG test items were declared valid, while for the sociomathematical norms questionnaire items, there were 24 items declared good and six items invalid. Next, each correct item is continued to be tested for reliability. The results of the reliability testing of the instrument are shown in Table 5.

Table 5. Reliability Test Results

Variable	Cronbach's alpha	Criteria	Conclusion
CUG	0.845	>0.700	Reliable
Sociomathematical Norm	0.917	>0.700	Reliable

Table 5 shows that all pre-service teachers' students' CUG test items were declared reliable. So, these two instruments can be used to collect research data.

Data Analysis

Furthermore, to deepen the contribution of DLM implementation to the achievement of students' CUG, interviews were conducted with 3 students who each represented high, medium and low sociomathematical norm categories. Respondents were given the code R1 for students with high sociomathematical norms, R2 for students with medium sociomathematical norms and R3 for students with low sociomathematical norms. The interview focused on how the contribution of DLM in learning has been carried out on the achievement of students' CUG based on the category of socio mathematical norms.

The research hypothesis was tested using a two-way ANOVA to test the effect of the learning model on the achievement of students' CUG based on the category of sociomathematical norms and the interactions between DLM and the achievement of students' CUG. Before the data were analyzed, the data achievement of students' CUG was carried out by prerequisite tests, namely the normality and homogeneity tests between DLM and CM classes.

RESULT AND DISCUSSION

The results and discussion sections present the results obtained and how they were achieved. The description must be comprehensive but still concise and coherent. Discussion of research results includes advantages and disadvantages, including testing. The results and discussion sections present the results obtained and how they were achieved. The description must be comprehensive but still concise and coherent. Discussion of research results includes advantages and disadvantages, including testing.

To test the hypotheses according to the research questions that have been submitted, a Two-Way ANOVA analysis was carried out to achieve students' CUG based on the category of sociomathematical norms. The description of the research data on the achievement of students' CUG is shown in Table 6.

Table 6. Data Description of Achievement of Pre-Service Teacher Student's CUG

Sociomathematical Norms Category	N	Learning Model	Mean	Std. Deviation
High	6	DLM	14,17	2,401
	5	CM	13,20	1,924
Medium	23	DLM	12,61	3,272
	23	CM	10,78	2,984
Low	6	DLM	10,50	3,391
	7	CM	8,00	3,367
Total	35	DLM	12,31	3,323
	35	CM	10,57	3,257

The prerequisite test is applied before testing the hypothesis. The prerequisites are the normality test using the Shapiro-Wilk test and the homogeneity test using the Lavene Statistic test. The normality test shows the achievement of students' CUG in DLM class, which shows Sig. = 0.384 > 0.05 with $\alpha = 0.05$. That is, it can be said that the achievement of students' CUG score data in DLM class is normally distributed. The normality test for the CM class yields a value of Sig.=0.928 > 0.05 with $\alpha = 0.05$, so it can be said that the achievement of students' CUG scores in the CM class is normally distributed.

Furthermore, the homogeneity test of the score data on CUG for the experimental and control classes used the Levene Statistic test. Based on the Levene Statistic test with α=0.05, Sig.= 0.603 was obtained. Because the value of Sig. = 0.603 > 0.05, it can be said that the achievement of students' CUG in DLM and CM classes has a homogeneous variance.

After the achievement, students' CUG data in each class is declared customarily distributed and has a homogeneous variant. The next step is a two-way ANOVA test with the results in Table 7.

Table 7. Results of Two-way Anova Test the achievement of students' CUG Based on Aspects of Sociomathematical Norms

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Model	44,956	1	44,956	4,454	,039

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Socimathematical Norms	79,814	2	39,907	3,954	,024
Model * Socimathematical Norms	10,742	2	5,371	,532	,049

Table 6 shows the Sig. in the model analysis is 0.039, less than 0.05 with α = 0.05. The research hypothesis states that there are differences in the achievement of CUG between students who implement DLM and students who carry out learning using the CM accepted. Table 4 shows that the average CUG who gets DLM is greater than CM, so it can be said that the achievement of students CUG who get DLM is better than that of students who get CM.

Table 5 shows the sociomathematical norm analysis at 0.024 <0.05 with α = 0.05, and the research hypothesis states that there are differences in the achievement of students' CUG between students who receive DLM and students who receive CM based on sociomathematical norm level. To find out which groups of students have different sociomathematical norm levels, proceed with the HSD Tukey test with results as shown in Table 8.

Table 8. Tukey's HSD Test Results on Data on Student's CUG on Sociomathematical Norm Level

(I)	(J)	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
High	High	2,22	,953	,059	-,07	4,51
	Medium	2,61*	1,000	,030	,21	5,01
Medium	High	-2,22	,953	,059	-4,51	,07
	Low	,39	,887	,898	-1,74	2,52
Low	High	-2,61*	1,000	,030	-5,01	-,21
	Medium	-,39	,887	,898	-2,52	1,74

Based on Table 6, there is a significant difference between the achievement of CUG in groups of students at high and low sociomathematical norm levels at α = 0.05. This shows a substantial difference in the achievement of students' CUG based on the level of sociomathematical norms.

Table 5 shows the analysis of the interaction between models and the sociomathematical norm of 0.049, and this value is less than 0.05 with a significant level of α = 0.05. The hypothesis states that interaction between DLM and students' sociomathematical norm factors influence the achievement of CUG, which is accepted.

The study results explain differences in learning towards achieving students' CUG based on aspects of sociomathematical norms. The statistical analysis showed that the average achievement of students' CUG in DLM was higher than that of CM students. These results indicate that DLM has a good influence on the achievement of students' CUG in plane geometry courses. These results corroborate research conducted by (Wardono et al., 2020), which found DLM to be more effective in mathematics than direct learning. In addition, the research revealed that applying DLM efficiently increased conceptual understanding skills (Suyitno et al., 2019) and activated learning in constructing interpersonal knowledge in understanding concepts (Permatasari et al., 2018).

Based on the results of statistical tests, it was confirmed that the average achievement of students' CUG in DLM as a whole is better than in CM. This also applies to the achievement of students' CUG in each category of student sociomathematical norms, although this only applies to high and low levels. So, these results indicate that DLM has a good influence on increasing students'

CUG in planned geometry courses. These findings align with research conducted by (Sánchez & García, 2014), which revealed that sociomathematical norms enable students to optimize mathematical arguments in each mathematics lesson to build concepts comprehensively.

The study results also show that students with high and low sociomathematical norms experience a good understanding of geometric concepts. These conditions indicate that the process of mathematical experience and explanation in the learning process to express arguments supports the achievement of CUG. This is corroborated by the research results of (Zembat et al., 2015); students who can explain a mathematical concept focus on describing problem-solving operations and understanding the concept being studied.

The study's results also show an interaction effect between DLM and students' sociomathematical norm factors on the achievement of CUG. Sociomathematical norm factors interact in learning geometry using DLM from CUG. Students with high and low sociomathematical abilities have good CUG, supported by the sociomathematical norm. This is in line with the results of research conducted by (Kang & Kim, 2016), which revealed that in the learning process, a student has a logical point of view about mathematical knowledge that will lead to a comprehensive understanding of concepts. Furthermore, Partanen & Kaasila (2015) found in their research that socimathematical norms can focus students on conceptual thoughts in solving a mathematical problem so that it can be adequately understood.

The positive contribution of DLM to the achievement of students' CUG is supposedly caused by the exploration process of using student worksheets as a guide in every learning process. Lecturers prepare student worksheets by aligning the syntax of DLM to assist students in understanding the geometric concepts presented in each plane geometry course so that student's ability to understand mathematical concepts can be improved. This can be seen in the researcher's interviews with students in the sociomathematical norm category while in the experimental class. Students were interviewed regarding the learning process carried out in plane geometry courses. The following are excerpts from interviews with researchers who were given the code P with R2 that was carried out:

P : *Do you think the learning [DLM] we have done was effective?*
R2 : *Very effective sir*
P : *What makes it effective?*
R2 : *Experience in other learning is only explained by the lecturer. But if the teaching has been done, I easily understand it. The questions on the worksheet led me to conclude, so I was more excited. So, better understand the concept, sir.*

Based on the interview excerpts, it can be seen that R2 feels that the learning uses DLM because the learning R2 feels guided by questions, so it concludes the intended concept. R2 also found it helpful to have student worksheets. Because the questions posed in it coherently help lead to the concept being studied. So that R2 better understands the concepts. This follows the opinion (of Baroody et al., 2015) describing the three main characteristics of discovery learning, namely: 1) exploring and solving problems and generalizing knowledge; 2) activities are based on stages and frequency; and 3) activities encouraging the integration of new knowledge into the existing knowledge base.

Second, the contribution from DLM is the group discussion process. The discussion process in DLM has an essential contribution to the formation of students' understanding of geometric concepts. In the discussion process, each student conveys their ideas. If ideas are wrong, other students straighten them out and provide input to the debate process. This process allows students in their groups to find the most considered correct ideas. As seen in the following interview excerpts with R3:

P : What is interesting in the learning [DLM] that has been done?
R3 : It was a fascinating group discussion, sir
P : What is interesting about the discussion process that has been carried out?
R3 : It is more fun, as all group members express their ideas. Even though something wasn't right, another friend explained it. So that we can understand the concept
P : Any difference of opinion?
R3 : There are differences of opinion about the answers. Although debating, in the end, the answer is considered the most correct

The interview results with R3 revealed differences of opinion within the group, but in the end, the argument that was considered correct was used. Thus, each group member can also understand the valid answers or ideas to avoid misunderstandings. This can also indicate that the discussion is very supportive of DLM because new ideas emerge in the discussion process from the consensus thoughts of each group member. In line with the opinion of (Tran et al., 2014), group discussions to solve problems together increase the possibility of new ideas from the thoughts of each group member. Even a student can get ideas from other students when he sees ideas and contexts of thinking about problems being solved by other students in a group (Saefulbahri, 2015).

Third, the contribution of MLD to the achievement of students' CUG is that the diversity of alternative solutions to problems strengthens students in making choices and analyzing the best solution to problems. Each student expresses their ideas by determining alternative solutions to geometric problems. The solutions offered by students in each group are different. The many alternative solutions enrich students' experience in understanding geometric concepts. Students analyze each solution provided in groups and conclude the best solution based on mutual agreement. The principle of cooperation in learning motivates the achievement of students' CUG. This is to the findings of (Hilliges et al., 2007), which revealed that collaboration in learning is needed to find several alternative solutions to make it easier for students because many ideas are presented. Furthermore, research conducted by (Graesser et al., 2018) concluded that collaboration between students creates a positive attitude and motivation in each learning process to solve mathematical problems.

Fourth, the contribution of MLD to the achievement of students' CUG is that classical presentation activities motivate students to understand concepts in each lesson. Learning to use DLM provides opportunities for students to express their ideas and present the results of the process. This can be shown from the interview excerpts with one of the students regarding the presentation process as follows.

P : Okay, let me repeat the advantages of the learning we have done [DLM].
R1 : In my opinion, the disadvantage is that there are group presentation activities, that's good, sir
P : Where is it good?

R1 : In the presentation activities, we can share with other groups, and there is also input from other groups. Usually, if my group has a wrong answer, we can ask the group presenting. So we can understand

Based on the interview excerpts, it was shown that R1 revealed that the advantage of the learning that was carried out was the presentation process because it could provide input from other groups from the results of their group discussions. DLM can activate students to explore geometry material by presenting the results of their discussions. This is corroborated by research (Purnami et al., 2018), which found that presentation activities in front of the class and training students' self-confidence can also control their ability to understand mathematical concepts. In addition, presentation activities also train students to communicate mathematical ideas in a structured manner according to logic (Maarif et al., 2020). Thus, the presentation process will strengthen the understanding of geometric concepts.

CONCLUSION

This study concludes first, CUG by using GDL is better than students who apply CM; second, there are differences in the achievement of CUG between students who apply DLM and students who apply CM based on socio-mathematical norms; third, there is the influence of the interaction between DLM and students' sociomathematical norm factors on the achievement of CUG; and fourth, the contribution of DLM implementation to student CUG achievement includes: (a) Student Worksheets with DLM syntax make it easier for students to deepen geometry material; (b) a discussion process that gives rise to students' freedom of expression; (c) the diversity of alternative problem solving strengthens students in determining the best choice and analysis of problem solving; (d) classical presentation activities motivate students to understand the concepts in each lesson.

Based on the conclusion, it can be said that DLM is effectively implemented in the lecture process, especially in geometry lectures. DLM is effective for forming students' ability to understand geometric concepts by considering the aspects of socio-mathematical norms. Students with high and low sociomathematical norms can adequately participate in learning with DLM to achieve students' understanding of geometric concepts.

This research recommends that DLM be applied effectively to develop students' CUG e, especially in geometry material. In the learning process, using DLM should optimize the group discussion process and create student worksheets by following the appropriate DLM steps. Furthermore, in learning geometry, it is necessary to pay attention to socio-mathematical norms to assist students in developing arguments in the discussion process to develop students' UGC.

ACKNOWLEDGEMENT

We would like to thank the parties who have supported the implementation of this research, especially in the Mathematics Education Study Program, Faculty of Teacher Training and Education, Universitas Muhammadiyah Prof. DR. HAMKA. All help and support is beneficial for the progress of the research we have carried out

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