## Analysis of TE (Transverse Electric)

## Modes of Symmetric Slab Waveguide

Harry Ramza<br>SPECTECH (Spectrum Technology) Research Group Department of Electrical, Electronic and Systems Engineering National University of Malaysia 43600 UKM-Bangi, Selangor, Malaysia hramza@eng.ukm.my

## Farshad Nasimi

SPECTECH (Spectrum Technology) Laboratory<br>Department of Electrical, Electronic and Systems Engineering<br>National University of Malaysia 43600 UKM-Bangi, Selangor, Malaysia

## Khairul Anuar Ishak

SPECTECH (Spectrum Technology) Laboratory
Department of Electrical, Electronic and Systems Engineering National University of Malaysia 43600 UKM-Bangi, Selangor, Malaysia

## Mohammad Syuhaimi Ab-Rahman

SPECTECH (Spectrum Technology) Laboratory
Department of Electrical, Electronic and Systems Engineering
National University of Malaysia 43600 UKM-Bangi, Selangor, Malaysia


#### Abstract

Description of integrated mode profile by determine of $\kappa, \gamma, \delta$ parameters as functions of the propagation constant $(\beta)$ and effective refractive index ( $\mathrm{n}_{\text {eff }}$ ). The profile can be seen from $\mathrm{E}(\mathrm{x})$ formula for each guide TE (Transverse Electric) modes. Assumptions given in this slab waveguide is used for wavelength $(\lambda) 1.55$ $\mu \mathrm{m}$, the thickness (d) of the core is $0.9 \mu \mathrm{~m}$ with a type of symmetric step-index slab waveguide, refractive index of $n_{1}$ is 3.5 and refractive index of $n_{2}$ is 3 , also $\mathrm{n}_{3}=\mathrm{n}_{1}$. The results of analysis are presented in graphical form by combining $\mathrm{TE}_{0}$ mode, $\mathrm{TE}_{1}$ mode and $\mathrm{TE}_{2}$ mode..


Keywords: Propagation constant, effective refractive index, slab waveguide, symmetric waveguide.

## 1 Introduction

The analysis of TE modes are started with the electric field polarized along y direction for a symmetric step index slab waveguide. This calculation is performed to determine the profile mode slab waveguides, and prove the characteristics of the TE mode that $n_{2}>n_{3}, n_{1}=n_{3}$ and the number of frequency normalization. A schematic diagram of a model for an symmetric slab waveguide is shown in Fig 1. The refractive index indices of the guiding layer, substrate and cover are $n_{g}, n_{s}$ and $n_{c}$ respectively. It's assumed that the refractive index of the substrate is greater than the cover.


Fig 1. A schematic of a symmetric step-index slab waveguide[1].
Depending on whether a total internal reflection occurs at the core-substrate or/and core-cover interfaces, there are at least three types of modes that may be supported by waveguide. They are guided modes, substrate radiation modes and superstrate-cover radiation modes as indicated in Fig 2 below.


Fig 2. The ray picture of mode on an-symmetric step index slab waveguide
(a) $\phi<\theta_{c}$ non-internal reflection condition, (b) $\phi<\theta_{c}$ non-internal reflection condition(c). $\phi>\theta_{c}$ internal reflection condition.

In Fig 2(a) shows that the light beams coming from substrate layer to guiding layer will occur light beams that came out on the cover layer, this incident is known as radiation modes [1]. In Fig 2(b) equal to 2(a), for this case, the incident light angle vanishingly small, or better known as leaky modes. In Fig 2 (c) shows that the light beam total internal reflection occurs.Text of section 1 .

## 2 Basic Theory

From Fig 1 and Fig 3, there are electric field (E) and magnetic field (H). Two field of type can performed into two Equations [2],

$$
\begin{equation*}
\bar{E}=\bar{i} E_{x}+\bar{j} E_{y}+\bar{k} E_{z} \tag{1}
\end{equation*}
$$

and

$$
\begin{equation*}
\bar{H}=\bar{i} H_{x}+\bar{j} H_{y}+\bar{k} H_{z} \tag{2}
\end{equation*}
$$

From Maxwell Equation, that;

$$
\begin{equation*}
\nabla \times \bar{E}=-\mu \frac{\partial H}{\partial t} \tag{3}
\end{equation*}
$$

Equation (3) can be expanded into [2],

$$
\begin{aligned}
& \left|\begin{array}{ccc}
\bar{i} & \bar{j} & \bar{k} \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
E_{x} & E_{y} & E_{z}
\end{array}\right|=-\mu \frac{\partial}{\partial t}\left[\bar{i} H_{x}+\bar{j} H_{y}+\bar{k} H_{z}\right] \\
& \quad \bar{i}\left[\frac{\partial E_{z}}{\partial y}-\frac{\partial E_{y}}{\partial z}\right]-\bar{j}\left[\frac{\partial E_{z}}{\partial x}-\frac{\partial E_{x}}{\partial z}\right]+\bar{k}\left[\frac{\partial E_{y}}{\partial x}-\frac{\partial E_{x}}{\partial z}\right]
\end{aligned}
$$

then,

$$
\begin{equation*}
=\bar{i}\left[-\mu \frac{\partial H_{x}}{\partial t}\right]+\bar{j}\left[-\mu \frac{\partial H_{y}}{\partial t}\right]+\bar{k}\left[-\mu \frac{\partial H_{z}}{\partial t}\right] \tag{4}
\end{equation*}
$$

Condition for slab waveguide is $\frac{\partial}{\partial y}=0$; therefore Equation (4) becomes [3],

$$
\begin{align*}
\frac{\partial E_{y}}{\partial z} & =\mu \frac{\partial H_{x}}{\partial t}  \tag{5}\\
\frac{\partial E_{z}}{\partial x}-\frac{\partial E_{x}}{\partial z} & =\mu \frac{\partial H_{y}}{\partial t}  \tag{6}\\
\frac{\partial E_{y}}{\partial x} & =-\mu \frac{\partial H_{z}}{\partial t} \tag{7}
\end{align*}
$$

As explained in Equation (3), by using Maxwell Equation below;

$$
\begin{equation*}
\nabla \times \bar{H}=\varepsilon \frac{\partial \bar{E}}{\partial t}=\varepsilon_{0} n^{2} \frac{\partial \bar{E}}{\partial t} \tag{8}
\end{equation*}
$$

from Equation (8) above, it is expanded into,

$$
\begin{align*}
& \left|\begin{array}{ccc}
\bar{i} & \bar{j} & \bar{k} \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
H_{x} & H_{y} & H_{z}
\end{array}\right|=-\varepsilon_{0} n^{2} \frac{\partial}{\partial t}\left[\bar{i} E_{x}+\bar{j} E_{y}+\bar{k} E_{z}\right] \\
& \quad \bar{i}\left[\frac{\partial H_{z}}{\partial y}-\frac{\partial H_{y}}{\partial z}\right]-\bar{j}\left[\frac{\partial H_{z}}{\partial x}-\frac{\partial H_{x}}{\partial z}\right]+\bar{k}\left[\frac{\partial H_{y}}{\partial x}-\frac{\partial H_{x}}{\partial z}\right]  \tag{9}\\
& \quad=\bar{i}\left[\varepsilon_{0} n^{2} \frac{\partial E_{x}}{\partial t}\right]+\bar{j}\left[\varepsilon_{0} n^{2} \frac{\partial E_{y}}{\partial t}\right]+\bar{k}\left[\varepsilon_{0} n^{2} \frac{\partial H_{z}}{\partial t}\right]
\end{align*}
$$

then, it shows that,

$$
\begin{equation*}
-\frac{\partial H_{y}}{\partial z}=\varepsilon_{0} n^{2} \frac{\partial E_{x}}{\partial t} \tag{10}
\end{equation*}
$$

$$
\begin{gather*}
\frac{\partial H_{x}}{\partial z}-\frac{\partial H_{z}}{\partial x}=\varepsilon_{0} n^{2} \frac{\partial E_{y}}{\partial t}  \tag{11}\\
\frac{\partial H_{y}}{\partial x}=\varepsilon_{0} n^{2} \frac{\partial E_{z}}{\partial t} \tag{12}
\end{gather*}
$$

## 3 TE (Transverse Electric) Mode



Fig 3. TE mode polarization [3].
Assume that is based on physical condition [1-4]

$$
\begin{align*}
& E_{y}=E_{y 0} e^{j(\omega t-\beta z)}  \tag{13}\\
& H_{x}=H_{x 0} e^{j(\omega t-\beta z)}  \tag{14}\\
& H_{z}=H_{z 0} e^{j(\omega t-\beta z)} \tag{15}
\end{align*}
$$

From Equations (13), (14) and (15). They can be performed using differential Equation,

$$
\begin{gathered}
\frac{\partial E_{y}}{\partial t}=j \omega \quad \text { and } \quad \frac{\partial E_{y}}{\partial z}=-j \beta \\
\frac{\partial H_{x}}{\partial t}=j \omega \quad \text { and } \quad \frac{\partial H_{x}}{\partial z}=-j \beta, \text { also } \\
\frac{\partial H_{z}}{\partial t}=j \omega \quad \text { and } \frac{\partial H_{z}}{\partial z}=-j \beta
\end{gathered}
$$

Main fields that worked in TE mode are $E_{y}, H_{x}$, and $H_{z}$ field. Therefore, Equation (5), (7) and (11) can be simplified into,

$$
\begin{array}{r}
-j \beta E_{y}=j \omega \mu H_{x} \\
\frac{\partial E_{y}}{\partial x}=-j \omega \mu H_{z} \\
-j \beta H_{x}-\frac{\partial H_{z}}{\partial x}=j \omega \varepsilon_{0} n^{2} E_{y} \tag{18}
\end{array}
$$

If Equations (16) and (17) are substituted into Equation (18), they can performed [3],

$$
\begin{equation*}
\frac{\partial^{2} E_{y}}{\partial x^{2}}+\left(k^{2} n^{2}-\beta\right) E_{y}=0 \tag{19}
\end{equation*}
$$

where, $k=\frac{\omega}{c}=\frac{\omega}{\sqrt{1 / \mu_{0} \varepsilon_{0}}}=\omega \sqrt{\mu_{0} \varepsilon_{0}}$
$k$ is free space wave number.
$\beta$ is called the propagation constant.
$n$ is called material refractive index.
Solution of differential Equation orde-2 of Equation (19) is

$$
\begin{equation*}
E_{y}=E_{y 01} e^{j\left(\sqrt{k^{2} n^{2}-\beta^{2}}\right) x}+E_{y 02} e^{j\left(\sqrt{k^{2} n^{2}-\beta^{2}}\right) x} \tag{20}
\end{equation*}
$$

or

$$
\begin{align*}
& E_{y}=A \cos \left(\sqrt{k^{2} n^{2}-\beta^{2}}\right) x \\
& +B \cos \left(\sqrt{k^{2} n^{2}-\beta^{2}}\right) x \tag{21}
\end{align*}
$$

i. For area $n_{\text {eff }}=n_{1}$ or cladding (superstrate) [4-9]:

Equation (20) can be changed to be:

$$
E_{y}=E_{y 011} e^{j\left(\sqrt{k^{2} n_{1}^{2}-\beta^{2}}\right) x}+E_{y 012} e^{j\left(\sqrt{k^{2} n_{1}^{2}-\beta^{2}}\right) x}
$$

from physical behavior is known that $E_{y} \rightarrow 0$; for $x \rightarrow \infty$. So,

$$
k^{2} n_{1}^{2}-\beta^{2}<0
$$

Solution of Equation can be

$$
\begin{equation*}
E_{y}=E_{y 01} e^{-\delta x} \tag{22}
\end{equation*}
$$

where,

$$
\begin{gather*}
E_{y 0}=E_{y 0011}+E_{y 012} \\
\delta=\sqrt{\beta^{2}-k^{2} n_{1}^{2}} \tag{23}
\end{gather*}
$$

$\delta$ is a positive real number.
ii. For area $n_{\text {eff }}=n_{2}$ or guiding (core) [4-9] :

Equation (20) can be changed to be:

$$
E_{y}=E_{y 021} e^{j\left(\sqrt{k^{2} n_{2}^{2}-\beta^{2}}\right) x}+E_{y 022} e^{-j\left(\sqrt{k^{2} n_{2}^{2}-\beta^{2}}\right) x}
$$

or

$$
E_{y}=A \cos \left(x \sqrt{k^{2} n_{2}^{2}-\beta^{2}}\right)+B \sin \left(x \sqrt{k^{2} n_{2}{ }^{2}-\beta^{2}}\right)
$$

Solution of Equation above to be,

$$
\begin{equation*}
E_{y}=E_{y 021} e^{j \omega x}+E_{y 022} e^{-j \kappa x} \tag{24}
\end{equation*}
$$

or

$$
\begin{equation*}
E_{y}=A \cos (\kappa x)+B \sin (\kappa x) \tag{25}
\end{equation*}
$$

where

$$
\begin{equation*}
\kappa=\sqrt{k^{2} n_{2}^{2}-\beta^{2}} \tag{26}
\end{equation*}
$$

$\kappa$ is a real number.
iii. For area $n_{\text {eff }}=n_{3}$ or substrate [4-9]:

Equation (20) can changed to be:

$$
\begin{equation*}
E_{y}=E_{y 031} e^{j\left(\sqrt{k^{2} n_{3}^{2}-\beta^{2}}\right) x}+E_{y 032 e^{j}}^{j\left(\sqrt{k^{2} n_{3}^{2}-\beta^{2}}\right) x} \tag{27}
\end{equation*}
$$

from physical behavior is known that $E_{y} \rightarrow 0$; for $x \rightarrow \infty$. So,

$$
k^{2} n_{3}^{2}-\beta^{2}<0
$$

Solution of Equation to be

$$
\begin{equation*}
E_{y}=E_{y 03} e^{\gamma x} \tag{28}
\end{equation*}
$$

Where,

$$
\begin{equation*}
\gamma=\sqrt{\beta^{2}-k^{2} n_{3}^{2}} \tag{29}
\end{equation*}
$$

$\gamma$ is a real number.

## 4 Calculation and Results

The assumption of this case is the wavelength ( $\lambda$ ) $1.55 \mu \mathrm{~m}$. Refractive index of guided layer $\left(\mathrm{n}_{2}\right)$ is 3.5 and refractive index of substrate layer $\left(\mathrm{n}_{3}\right)$ and cover layer $\left(\mathrm{n}_{1}\right)$ are 3.00 . In the Fig 4 shows that the frequency of normalization or $V$-parameter obtained is [7-11]

$$
\begin{align*}
V & =2 \pi\left(\frac{d}{\lambda}\right) \sqrt{n_{1}^{2}-n_{2}^{2}}  \tag{30}\\
V & =3.289
\end{align*}
$$

The value above is obtained from $\mathrm{d}=0.45$. In Fig 4 below, $V$ - value of is shown on the dashed line.


Fig 4. Characteristic Equation diagram TE Modes.

In the Fig 4 above showed that the solid line represent the graph of the even TE modes and the dash-dot line represent the graph of the odd - TE modes [12]. Based on the Fig 4, the first confined mode is identified to be at the value of $\kappa d \leq 1.198$ while the second confined mode is identified to be in the range of $2.34693<\kappa d \leq 3.26396$.
Basically at the specific value of confined mode ( $\kappa d$ ), the parameters of the equation could be defined by with the value below using the above.

Table 1. Confined mode calculation.

| $\kappa$ rd | 1.19800 | 2.34693 | 3.26396 |
| :--- | :---: | :---: | :---: |
| $V$-parameter | $\mathbf{3 . 0 6 3 0 0}$ | $\mathbf{2 . 3 0 4 0 0}$ | $\mathbf{0 . 4 0 1 0 0}$ |
| even TE modes | $\mathbf{3 . 0 6 3 0 0}$ | -2.39100 | $\mathbf{0 . 4 0 1 0 0}$ |
| odd TE modes | -0.46900 | $\mathbf{2 . 3 0 4 0 0}$ | -26.53200 |



Fig 5. Mode profile for $\mathrm{TE}_{0}, \mathrm{TE}_{1}$ dan $\mathrm{TE}_{2}$.
In the Fig 5 shows the mode profile in the slab waveguide. Profile is obtained from the Equation $\mathrm{E}_{\mathrm{y}}(\mathrm{x})$ on the ordinate axis and the waveguide layer $\mathrm{x}_{1}$ (substrate), $\mathrm{x}_{2}$ (guided) and $\mathrm{x}_{3}$ (cover) on the abscissa axis. $\mathrm{TE}_{0}$ values that must be met are:

$$
\begin{equation*}
\tan \left(\kappa d_{0}\right)=\frac{\sqrt{V^{2}-\kappa d_{0}}}{\kappa d_{0}} \tag{31}
\end{equation*}
$$

where $\kappa d_{0}$ is 1.3 , then the angle of $\kappa d_{0}$ is $1.1980^{\circ}$.

$$
k_{0}=\frac{\kappa d_{0}}{d}=2.662
$$

The results above will be used to determine the propagation constants, namely:

$$
\begin{equation*}
\beta_{0}=\sqrt{\left(\frac{2 \pi}{\lambda}\right)^{2} n_{1}^{2}-\left(\frac{\kappa d_{0}}{d}\right)^{2}} \tag{32}
\end{equation*}
$$

$$
\begin{equation*}
\gamma_{0}=\sqrt{\beta_{0}^{2}-\left(\frac{2 \pi}{\lambda}\right)^{2} n_{2}^{2}} \tag{33}
\end{equation*}
$$

Equation (32) and (33) will yield a value of $\beta_{0}=13.936$ and $\gamma_{0}=6.806$. From Equation (34), will get the value of effective refractive index $\left(n_{\text {eff }, 0}\right)$,

$$
\begin{align*}
n_{e f f, 0} & =\beta_{0} \frac{\lambda}{2 \pi}  \tag{34}\\
n_{\text {eff }, 0} & =3.438
\end{align*}
$$

For $\mathrm{TE}_{1}$ values that must be met are:

$$
\begin{equation*}
\cot \left(\kappa d_{1}\right)=-\frac{\sqrt{V^{2}-\kappa d_{1}}}{\kappa d_{1}} \tag{35}
\end{equation*}
$$

Same as the above case $\kappa d_{1}=2.00$, then the angle of $\kappa d_{1}$ is $2.347^{\circ}$. For the value $k_{1}=5.215, \beta_{1}=13.195, \gamma_{1}=5.119$ and $n_{\text {eff }, 1}=3.255$. For $\mathrm{TE}_{2}$ values that must be met are :

$$
\begin{equation*}
\tan \left(k d_{2}\right)=-\frac{\sqrt{V^{2}-k d_{2}}}{k d_{2}} \tag{36}
\end{equation*}
$$

for for $\kappa d_{2}=3.00$, then the angle of $\kappa d_{2}=3.264^{0}$. Therefore $k_{2}=7.253, \beta_{2}=12.194, \gamma_{2}=0.892$ and $n_{\text {eff }, 2}=3.008$.

## 5 Conclusion

We found that the mode profiles is shown by $\mathrm{TE}_{0} \mathrm{TE}_{1}$ and $\mathrm{TE}_{2}$. $V$-parameter or normalized frequency is 3.289 . Boundary condition of mode value on the each layer are $-0.9 \leq x_{1}<-0.45$ for substrate layer, $-0.45 \leq x_{2} \leq 0.45$ for guided layer and $0.45<\mathrm{x}_{3} \leq 0.9$ for cover layer. TE0 TE1 and TE2 as the mode profile that was calculated. Simulated quantization value is 0.01 . Effective refraction index of material on substrate layer $\left(n_{\text {eff }, 0}\right)$ is 3.438 for $\mathrm{TE}_{0}$, effective refractive index on guided layer $\left(n_{\text {eff }, 1}\right)$ is 3.255 for $\mathrm{TE}_{1}$ and effective refractive index on cover layer $\left(n_{\text {eff }, 2}\right)$ is 3.008 for $\mathrm{TE}_{2}$.

## Acknowledgments

This work is sponsored by Research University Grant from Universiti Kebangsaan Malaysia, Bangi, Selangor Darul Ehsan with code number UKM-GUP-2011-048.
H. R author wish to thank Assoc. Prof. Dr. Akhiruddin Maddu from Department Physic from Bogor Agriculture of Institute and Dr. Ary Syahriar DIC from the

Indonesia Agency for The Assessment and Application of Technology for their support and encouragement.

## References

[1] Cherin A H. An introduction to optical fibers: McGraw-Hill, 1983.
[2] Keiser G. Optical Fiber Communications: McGraw-Hill Companies, 2010.
[3] Lee D L. Electromagnetic principles of integrated optics: Wiley, 1986.
[4] Snyder A W. and Love J. Optical waveguide theory. New York: Springer Verlag, 2007.
[5] Syms R R A. and Cozens J R. Optical guided waves and devices: McGraw-Hill, 1992.
[6] Kogelnik H. Theory of dielectric waveguides. in Integrated optics, T. Tamir, Ed., ed Berlin: Springer, 1979.
[7] Kasap S O. Optoelectronics and photonics: principles and practices: Prentice Hall, 2001.
[8] Khorasani S. and Rashidian B, 2001. Guided light propagation in dielectric slab waveguide with conducting interfaces. J. Opt. A. Pure Appl. Opt. 3: pp. 380-386.
[9] Yariv A. Optical electronics in modern communications. Oxford: Oxford University Press, 1997.
[10] Adams M J. An introduction to optical waveguides. New York: John Wiley \& Sons, 1981.
[11] Calvo M L. and Lakshminarayanan V. Optical waveguides: from theory to applied technologies: CRC Press, 2007.
[12] Iizuka K. Elements of Photonics: In free space and special media: Wiley-Interscience, 2002.

## Appendix

A. 1. Slab - Waveguide Analysis


We assumed that cladding $\gg 2 \mathrm{a}$, with $\mathrm{x}_{2}$ and $\mathrm{x}_{3}$ is the width and length of the slab-waveguide. The two conditions for wave to propagate are :

1. $\gamma^{2}>0$ shows that wave propagate through the core.
2. $\gamma^{2}<0$ shows that there is no wave propagation through the cladding.

Continuity boundary condition,

$$
\begin{align*}
& \hat{n} \times \hat{E}_{1}=\hat{n} \times \hat{E}_{2}  \tag{a.1}\\
& \hat{n} \times \hat{H}_{1}=\hat{n} \times \hat{H}_{2} \tag{a.2}
\end{align*}
$$

with the time and $x_{3}$ dependence

$$
\begin{equation*}
e^{j\left(\omega t-\beta x_{3}\right)} \tag{a.3}
\end{equation*}
$$

The component $E_{y}$ is obtained as solution of the reduce wave equation [13]

$$
\begin{equation*}
\frac{\partial^{2} E_{y}}{\partial x^{2}}+a^{2}=0 \tag{a.4}
\end{equation*}
$$

where,

$$
\begin{array}{ll}
a>0, & E_{y}=A \cos (a x)+B \sin (a x) \\
a<0, & E_{y}=A e^{a x} \tag{a.6}
\end{array}
$$

For TE mode can be written wave equation,

$$
\begin{equation*}
\bar{E}\left(x_{1}, x_{2}, x_{3}, t\right)=\hat{y} E\left(x_{1}, x_{2}\right) e^{\left[j\left(\omega t-\beta x_{3}\right)\right]} \tag{a.7}
\end{equation*}
$$

$E$ in direction of $x_{2}$ is unlimited uniform value, $E$ is only vary with x and is expressed as;

$$
\begin{equation*}
\left(\frac{\partial^{2}}{\partial x^{2}}+\gamma^{2}\right) E\left(x_{1}\right)=0 \tag{a.8}
\end{equation*}
$$

therefore the solution of Eigen value can be written as ;
core :

$$
\begin{equation*}
E_{1}\left(x_{1}\right)=A \cos \left(2 x_{1}\right)+B \sin \left(2 x_{1}\right),-a \leq x_{1} \leq a \tag{a.9}
\end{equation*}
$$

cladding:

$$
\begin{array}{ll}
E_{2}\left(x_{1}\right)=c e^{\left(-a x_{1}\right)} & x_{1} \geq a \\
E_{2}\left(x_{1}\right)=d e^{\left(+a x_{1}\right)} & x_{1} \leq-a \tag{a.11}
\end{array}
$$

with,
core :

$$
\begin{equation*}
\gamma^{2}=k_{1}^{2}-\beta^{2}=\omega^{2} \mu_{1} \varepsilon_{1}-\beta^{2}=n_{1}^{2} k_{0}^{2}-\beta^{2} \tag{a.12}
\end{equation*}
$$

Cladding:

$$
\begin{equation*}
\alpha^{2}=\beta^{2}-k_{2}{ }^{2}=\beta^{2}-\omega^{2} \mu_{2} \varepsilon_{2}=\beta^{2}-n_{2}{ }^{2} k_{0}{ }^{2} \tag{a.13}
\end{equation*}
$$

Boundary condition $x=a$
Continuity equation is $E_{1}\left(x_{1}\right)=E_{2}\left(x_{1}\right)$, then
if
then

$$
\begin{equation*}
-\gamma A \sin (\gamma a)+\gamma B \cos (\gamma a)=-\alpha c e^{(-\alpha a)} \tag{a.16}
\end{equation*}
$$

Boundary condition $x=-a$
Continuity equation is $E_{1}\left(x_{1}\right)=E_{2}\left(x_{1}\right)$, then
if

$$
\begin{gather*}
A \cos (\gamma a)-B \sin (\gamma a)=d e^{(-\alpha a)}  \tag{a.17}\\
\frac{\partial E_{1}\left(x_{1}\right)}{\partial x_{1}}=\frac{\partial E_{2}\left(x_{1}\right)}{\partial x_{1}}
\end{gather*}
$$

then

$$
\begin{equation*}
\gamma A \sin (\gamma a)+\gamma B \cos (\gamma a)=-\alpha d e^{(-\alpha a)} \tag{a.18}
\end{equation*}
$$

Substitute eq (a. 14) and (a. 17),

$$
\begin{aligned}
& A \cos (\gamma a)+B \sin (\gamma a)=c e^{(-\alpha a)} \\
& A \cos (\gamma a)-B \sin (\gamma a)=d e^{(-\alpha a)}
\end{aligned}
$$

Adding above equation,

$$
\begin{equation*}
2 A \cos (\gamma a)=(c+d) e^{(-\alpha a)} \tag{a.19}
\end{equation*}
$$

Substitute eq (a. 16) and (a. 18),

$$
\begin{aligned}
& -\gamma A \sin (\gamma a)+\gamma B \cos (\gamma a)=-\alpha c e^{(-\alpha a)} \\
& \gamma A \sin (\gamma a)+\gamma B \cos (\gamma a)=-\alpha d e^{(-\alpha a)}
\end{aligned}
$$

Subtracting above equation,

$$
\begin{equation*}
2 \gamma A \sin (\gamma a)=\alpha(\mathrm{c}+d) e^{(-\alpha a)} \tag{a.20}
\end{equation*}
$$

We can divide eq (a. 20) and (a.19)

$$
\begin{equation*}
\frac{2 \gamma A \sin (\gamma a)}{2 A \cos (\gamma a)}=\frac{\alpha(\mathrm{c}+d) e^{(-\alpha a)}}{(c+d) e^{(-\alpha a)}} \tag{a.21}
\end{equation*}
$$

then,

$$
\begin{equation*}
\tan (\gamma a)=\frac{\alpha}{\gamma} \tag{a.22}
\end{equation*}
$$

where $a=\frac{h}{2}$
In complete equation can be written,

$$
\begin{equation*}
\tan \left(\left(n_{1}{ }^{2} k_{0}^{2}-n_{\text {eff }}{ }^{2} k_{0}{ }^{2} 2\right) \frac{h}{2}\right)-\left(\frac{n_{\text {eff }}{ }^{2} k_{0}{ }^{2}-n_{2}{ }^{2} k_{0}{ }^{2}}{n_{1}{ }^{2} k_{0}{ }^{2}-n_{\text {eff }}{ }^{2} k_{0}{ }^{2}}\right)=0 \tag{a.23}
\end{equation*}
$$

using the numerical method of the equation above, the effective value of refractive index could be determined.
A. 2. MATLAB Programming.
\%\%\%\%\%\%\%\%\%\%\%\%\%\%\%\%\%\%\%\%\%\%\%\%
\%Bisection program
function bisect(f,h,a,b)
tol $=0.0000000001$;
$\mathrm{fa}=\mathrm{feval}(\mathrm{f}, \mathrm{h}, \mathrm{a})$;
$\mathrm{fb}=$ feval (f, h, b);
if ( $\mathrm{tol}<=0$ )
fprintf('tol should be positive number\n');
return
end
\%\%\%\%\%\%\%\%\%\%\%\%\%\%\%\%\%\%

```
if (fa*fb > 0)
    fprintf('Input a and b are out of interval\n');
    else
while 1
    if (abs (b-a) <= tol)
    break
end
    c}=(\textrm{a}+\textrm{b})/2
    fc = feval(f,h,c);
%%%%%%%%%%%%%%%%%%%
    If (c==a | c==b)
    fprintf ('maximum possible precission achieved\n');
    break
end
%%%%%%%%%%%%%%%%%%%
    if (fa*fc > 0)
        a=c;
        fa=fc;
        else
            b = c;
            fb = fc;
            end
    end
    fprintf ('Neffective value = %18.9f\n', b);
end
%%%%%%%%%%%%%%%%%%%%%%%%
%Execution program
function y=f(h/x)
y=tan((h*pi/1.55)*sqrt(1.468^2-x.^2)) -
    sqrt(x.^2-1.458^2))/(sqrt(1.468^2-x.^2));
%%%%%%%%%%%%%%%%%%%%%%%%
```

Received: September, 2012

