Analysis of TE (Transverse Electric)

Modes of Symmetric Slab Waveguide

Harry Ramza

SPECTECH (Spectrum Technology) Research Group Department of Electrical, Electronic and Systems Engineering National University of Malaysia 43600 UKM-Bangi, Selangor, Malaysia hramza@eng.ukm.my

Farshad Nasimi

SPECTECH (Spectrum Technology) Laboratory Department of Electrical, Electronic and Systems Engineering National University of Malaysia 43600 UKM-Bangi, Selangor, Malaysia

Khairul Anuar Ishak

SPECTECH (Spectrum Technology) Laboratory Department of Electrical, Electronic and Systems Engineering National University of Malaysia 43600 UKM-Bangi, Selangor, Malaysia

Mohammad Syuhaimi Ab-Rahman

SPECTECH (Spectrum Technology) Laboratory Department of Electrical, Electronic and Systems Engineering National University of Malaysia 43600 UKM-Bangi, Selangor, Malaysia

Abstract

Description of integrated mode profile by determine of κ , γ , δ parameters as functions of the propagation constant (β) and effective refractive index (n_{eff}). The profile can be seen from E(x) formula for each guide TE (Transverse Electric) modes. Assumptions given in this slab waveguide is used for wavelength (λ) 1.55 µm, the thickness (d) of the core is 0.9 µm with a type of symmetric step-index slab waveguide, refractive index of n_1 is 3.5 and refractive index of n_2 is 3, also $n_3 = n_1$. The results of analysis are presented in graphical form by combining TE₀ mode, TE₁ mode and TE₂ mode.

Keywords: Propagation constant, effective refractive index, slab waveguide, symmetric waveguide.

1 Introduction

The analysis of TE modes are started with the electric field polarized along y direction for a symmetric step index slab waveguide. This calculation is performed to determine the profile mode slab waveguides, and prove the characteristics of the TE mode that $n_2 > n_3$, $n_1 = n_3$ and the number of frequency normalization. A schematic diagram of a model for an symmetric slab waveguide is shown in Fig 1. The refractive index indices of the guiding layer, substrate and cover are n_g , n_s and n_c respectively. It's assumed that the refractive index of the substrate is greater than the cover.



Fig 1. A schematic of a symmetric step-index slab waveguide[1].

Depending on whether a total internal reflection occurs at the core-substrate or/and core-cover interfaces, there are at least three types of modes that may be supported by waveguide. They are guided modes, substrate radiation modes and superstrate-cover radiation modes as indicated in Fig 2 below.





In Fig 2(a) shows that the light beams coming from substrate layer to guiding layer will occur light beams that came out on the cover layer, this incident is known as radiation modes [1]. In Fig 2(b) equal to 2(a), for this case, the incident light angle vanishingly small, or better known as leaky modes. In Fig 2 (c) shows that the light beam total internal reflection occurs.Text of section 1.

2 Basic Theory

From Fig 1 and Fig 3, there are electric field (E) and magnetic field (H). Two field of type can performed into two Equations [2],

$$\overline{E} = \overline{i}E_x + \overline{j}E_y + \overline{k}E_z \tag{1}$$

and

$$\overline{H} = \overline{i}H_x + \overline{j}H_y + \overline{k}H_z \tag{2}$$

From Maxwell Equation, that;

$$\nabla \times \overline{E} = -\mu \frac{\partial H}{\partial t} \tag{3}$$

Equation (3) can be expanded into [2],

$$\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -\mu \frac{\partial}{\partial t} \left[\bar{i}H_x + \bar{j}H_y + \bar{k}H_z \right] \bar{i} \left[\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right] - \bar{j} \left[\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} \right] + \bar{k} \left[\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial z} \right] = \bar{i} \left[-\mu \frac{\partial H_x}{\partial t} \right] + \bar{j} \left[-\mu \frac{\partial H_y}{\partial t} \right] + \bar{k} \left[-\mu \frac{\partial H_z}{\partial t} \right]$$
(4)

then,

Condition for slab waveguide is $\frac{\partial}{\partial y} = 0$; therefore Equation (4) becomes [3],

$$\frac{\partial E_{y}}{\partial z} = \mu \frac{\partial H_{x}}{\partial t}$$
(5)

$$\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} = \mu \frac{\partial H_y}{\partial t}$$
(6)

$$\frac{\partial E_{y}}{\partial x} = -\mu \frac{\partial H_{z}}{\partial t}$$
(7)

As explained in Equation (3), by using Maxwell Equation below;

$$\nabla \times \overline{H} = \varepsilon \frac{\partial \overline{E}}{\partial t} = \varepsilon_0 n^2 \frac{\partial \overline{E}}{\partial t}$$
(8)

from Equation (8) above, it is expanded into, $|\overline{z}, \overline{z}|$

$$\begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = -\varepsilon_0 n^2 \frac{\partial}{\partial t} \left[\overline{i}E_x + \overline{j}E_y + \overline{k}E_z \right]$$
$$\overline{i} \left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right] - \overline{j} \left[\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z} \right] + \overline{k} \left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial z} \right]$$
$$= \overline{i} \left[\varepsilon_0 n^2 \frac{\partial E_x}{\partial t} \right] + \overline{j} \left[\varepsilon_0 n^2 \frac{\partial E_y}{\partial t} \right] + \overline{k} \left[\varepsilon_0 n^2 \frac{\partial H_z}{\partial t} \right]$$
(9)

then, it shows that,

$$-\frac{\partial H_{y}}{\partial z} = \varepsilon_{0} n^{2} \frac{\partial E_{x}}{\partial t}$$
(10)

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = \mathcal{E}_0 n^2 \frac{\partial E_y}{\partial t}$$
(11)

$$\frac{\partial H_y}{\partial x} = \varepsilon_0 n^2 \frac{\partial E_z}{\partial t}$$
(12)

3 TE (Transverse Electric) Mode



Fig 3. TE mode polarization [3].

Assume that is based on physical condition [1-4] $E_{y} = E_{y0}e^{j(\omega t - \beta z)}$ (13)

$$H_x = H_{x0} e^{j(\omega t - \beta z)} \tag{14}$$

$$H_z = H_{z0} e^{j(\omega t - \beta z)} \tag{15}$$

From Equations (13), (14) and (15). They can be performed using differential Equation,

$$\frac{\partial E_{y}}{\partial t} = j\omega \text{ and } \frac{\partial E_{y}}{\partial z} = -j\beta$$
$$\frac{\partial H_{x}}{\partial t} = j\omega \text{ and } \frac{\partial H_{x}}{\partial z} = -j\beta \text{ , also}$$
$$\frac{\partial H_{z}}{\partial t} = j\omega \text{ and } \frac{\partial H_{z}}{\partial z} = -j\beta$$

Main fields that worked in TE mode are E_y , H_x , and H_z field. Therefore, Equation (5), (7) and (11) can be simplified into,

$$-j\beta E_{y} = j\omega\mu H_{x}$$
(16)

$$\frac{\partial E_{y}}{\partial x} = -j\omega\mu H_{z} \tag{17}$$

$$-j\beta H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon_0 n^2 E_y$$
(18)

If Equations (16) and (17) are substituted into Equation (18), they can performed [3],

$$\frac{\partial^2 E_y}{\partial x^2} + (k^2 n^2 - \beta) E_y = 0$$
(19)
where, $k = \frac{\omega}{c} = \frac{\omega}{\sqrt{\frac{1}{\mu_0 \varepsilon_0}}} = \omega \sqrt{\mu_0 \varepsilon_0}$

k is free space wave number.

 β is called the propagation constant.

n is called material refractive index.

Solution of differential Equation orde-2 of Equation (19) is

$$E_{y} = E_{y01} e^{j\left(\sqrt{k^{2}n^{2} - \beta^{2}}\right)x} + E_{y02} e^{j\left(\sqrt{k^{2}n^{2} - \beta^{2}}\right)x}$$
(20)

or

$$E_{y} = A\cos\left(\sqrt{k^{2}n^{2} - \beta^{2}}\right)x + B\cos\left(\sqrt{k^{2}n^{2} - \beta^{2}}\right)x$$
(21)

i. For area $n_{eff} = n_1$ or cladding (superstrate) [4-9]:

Equation (20) can be changed to be:

$$E_{y} = E_{y011}e^{j\left(\sqrt{k^{2}n_{1}^{2}-\beta^{2}}\right)x} + E_{y012}e^{j\left(\sqrt{k^{2}n_{1}^{2}-\beta^{2}}\right)x}$$

from physical behavior is known that $E_y \rightarrow 0$; for $x \rightarrow \infty$. So,

$$k^2 n_1^2 - \beta^2 < 0$$

Solution of Equation can be

$$E_{y} = E_{y01} e^{-\delta x} \tag{22}$$

where,

$$E_{y0} = E_{y011} + E_{y012}$$

$$\delta = \sqrt{\beta^2 - k^2 n_1^2}$$
(23)

 δ is a positive real number.

ii. For area $n_{eff} = n_2$ or guiding (core) [4-9] : Equation (20) can be changed to be:

$$E_{y} = E_{y021}e^{j\left(\sqrt{k^{2}n_{2}^{2}-\beta^{2}}\right)x} + E_{y022}e^{-j\left(\sqrt{k^{2}n_{2}^{2}-\beta^{2}}\right)x}$$

or

$$E_{y} = A\cos\left(x\sqrt{k^{2}n_{2}^{2} - \beta^{2}}\right) + B\sin\left(x\sqrt{k^{2}n_{2}^{2} - \beta^{2}}\right)$$

Solution of Equation above to be,

$$E_{y} = E_{y021}e^{j\kappa x} + E_{y022}e^{-j\kappa x}$$
(24)

or

$$E_{y} = A\cos(\kappa x) + B\sin(\kappa x)$$
(25)

where

$$\kappa = \sqrt{k^2 n_2^2 - \beta^2} \tag{26}$$

 κ is a real number.

iii. For area $n_{eff} = n_3$ or substrate [4-9]:

Equation (20) can changed to be:

$$E_{y} = E_{y031}e^{j\left(\sqrt{k^{2}n_{3}^{2} - \beta^{2}}\right)x} + E_{y032}e^{j\left(\sqrt{k^{2}n_{3}^{2} - \beta^{2}}\right)x}$$
(27)

from physical behavior is known that $E_{y} \rightarrow 0$; for $x \rightarrow \infty$. So,

Where,

$$E_{y} = E_{y03} e^{\gamma x}$$
(28)

$$\gamma = \sqrt{\beta^2 - k^2 n_3^2} \tag{29}$$

 γ is a real number.

4 Calculation and Results

The assumption of this case is the wavelength (λ) 1.55 μ m. Refractive index of guided layer (n_2) is 3.5 and refractive index of substrate layer (n_3) and cover layer (n_1) are 3.00. In the Fig 4 shows that the frequency of normalization or *V*-parameter obtained is [7-11]

 $k^2 n_3^2 - \beta^2 < 0$

$$V = 2\pi \left(\frac{d}{\lambda}\right) \sqrt{n_1^2 - n_2^2}$$

$$V = 3.289$$
(30)

The value above is obtained from d = 0.45. In Fig 4 below, V - value of is shown on the dashed line.

(28)



Fig 4. Characteristic Equation diagram TE Modes.

In the Fig 4 above showed that the solid line represent the graph of the even - TE modes and the dash-dot line represent the graph of the odd - TE modes [12]. Based on the Fig 4, the first confined mode is identified to be at the value of $\kappa d \le 1.198$ while the second confined mode is identified to be in the range of $2.34693 < \kappa d \le 3.26396$.

Basically at the specific value of confined mode (κd), the parameters of the equation could be defined by with the value below using the above.

кd	1.19800	2.34693	3.26396
V-parameter	3.06300	2.30400	0.40100
even TE modes	3.06300	-2.39100	0.40100
odd TE modes	-0.46900	2.30400	-26.53200

Table 1. Confined mode calculation.



Fig 5. Mode profile for TE_0 , TE_1 dan TE_2 .

In the Fig 5 shows the mode profile in the slab waveguide. Profile is obtained from the Equation $E_y(x)$ on the ordinate axis and the waveguide layer x_1 (substrate), x_2 (guided) and x_3 (cover) on the abscissa axis. TE₀ values that must be met are:

$$\tan(\kappa d_0) = \frac{\sqrt{V^2 - \kappa d_0}}{\kappa d_0}$$
(31)

where κd_0 is 1.3, then the angle of κd_0 is 1.1980⁰.

$$k_0 = \frac{\kappa d_0}{d} = 2.662$$

The results above will be used to determine the propagation constants, namely:

$$\beta_0 = \sqrt{\left(\frac{2\pi}{\lambda}\right)^2 n_1^2 - \left(\frac{\kappa d_0}{d}\right)^2}$$
(32)

$$\gamma_0 = \sqrt{\beta_0^2 - \left(\frac{2\pi}{\lambda}\right)^2 n_2^2} \tag{33}$$

Equation (32) and (33) will yield a value of $\beta_0 = 13.936$ and $\gamma_0 = 6.806$. From Equation (34), will get the value of effective refractive index $(n_{eff,0})$,

$$n_{eff,0} = \beta_0 \frac{\lambda}{2\pi}$$

$$n_{eff,0} = 3.438$$
(34)

For TE₁ values that must be met are:

$$\cot(\kappa d_1) = -\frac{\sqrt{V^2 - \kappa d_1}}{\kappa d_1} \tag{35}$$

Same as the above case $\kappa d_1 = 2.00$, then the angle of κd_1 is 2.347⁰. For the value $k_1 = 5.215$, $\beta_1 = 13.195$, $\gamma_1 = 5.119$ and $n_{eff,1} = 3.255$. For TE₂ values that must be met are :

$$\tan(kd_2) = -\frac{\sqrt{V^2 - kd_2}}{kd_2}$$
(36)

for for $\kappa d_2 = 3.00$, then the angle of $\kappa d_2 = 3.264^{\circ}$. Therefore $k_2 = 7.253$, $\beta_2 = 12.194$, $\gamma_2 = 0.892$ and $n_{eff,2} = 3.008$.

5 Conclusion

We found that the mode profiles is shown by TE₀ TE₁ and TE₂. *V*-parameter or normalized frequency is 3.289. Boundary condition of mode value on the each layer are $-0.9 \le x_1 < -0.45$ for substrate layer, $-0.45 \le x_2 \le 0.45$ for guided layer and $0.45 < x_3 \le 0.9$ for cover layer. TE0 TE1 and TE2 as the mode profile that was calculated. Simulated quantization value is 0.01. Effective refraction index of material on substrate layer ($n_{eff,0}$) is 3.438 for TE₀, effective refractive index on guided layer ($n_{eff,1}$) is 3.255 for TE₁ and effective refractive index on cover layer ($n_{eff,2}$) is 3.008 for TE₂.

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Appendix

A. 1. Slab – Waveguide Analysis



We assumed that cladding >> 2a, with x_2 and x_3 is the width and length of the slab-waveguide. The two conditions for wave to propagate are :

- 1. $\gamma^2 > 0$ shows that wave propagate through the core.
- 2. $\gamma^2 < 0$ shows that there is no wave propagation through the cladding.

Continuity boundary condition,

$$\hat{n} \times \hat{E}_1 = \hat{n} \times \hat{E}_2 \tag{a. 1}$$

$$\hat{n} \times \hat{H}_1 = \hat{n} \times \hat{H}_2 \tag{a. 2}$$

with the time and x_3 dependence

$$e^{j(\omega t - \beta x_3)}$$
 (a. 3)

The component E_y is obtained as solution of the reduce wave equation [13]

$$\frac{\partial^2 E_y}{\partial x^2} + a^2 = 0 \tag{a. 4}$$

where,

$$a > 0, \quad E_y = A\cos(ax) + B\sin(ax)$$
 (a. 5)

$$a < 0, \qquad E_y = Ae^{ax} \tag{a. 6}$$

For TE mode can be written wave equation,

$$\overline{E}(x_1, x_2, x_3, t) = \hat{y} E(x_1, x_2) e^{[j(\omega t - \beta x_3)]}$$
(a. 7)

E in direction of x_2 is unlimited uniform value, *E* is only vary with x and is expressed as;

$$\left(\frac{\partial^2}{\partial x^2} + \gamma^2\right) E(x_1) = 0$$
 (a. 8)

therefore the solution of Eigen value can be written as ; core :

$$E_1(x_1) = A\cos(yx_1) + B\sin(yx_1), \ -a \le x_1 \le a$$
 (a. 9)

cladding:

$$E_2(x_1) = ce^{(-\alpha x_1)}$$
 $x_1 \ge a$ (a. 10)

$$E_2(x_1) = de^{(+\alpha x_1)}$$
 $x_1 \le -a$ (a. 11)

with,

core :
$$\gamma^2 = k_1^2 - \beta^2 = \omega^2 \mu_1 \varepsilon_1 - \beta^2 = n_1^2 k_0^2 - \beta^2$$
 (a. 12)

Cladding:
$$\alpha^2 = \beta^2 - k_2^2 = \beta^2 - \omega^2 \mu_2 \varepsilon_2 = \beta^2 - n_2^2 k_0^2$$
 (a. 13)

Boundary condition x = aContinuity equation is $E_1(x_1) = E_2(x_1)$, then

$$A\cos(\gamma a) + B\sin(\gamma a) = c e^{(-\alpha a)}$$
(a. 14)

if
$$\frac{\partial E_1(x_1)}{\partial x_1} = \frac{\partial E_2(x_1)}{\partial x_1}$$
 (a. 15)

then

$$-\gamma A \sin(\gamma a) + \gamma B \cos(\gamma a) = -\alpha \ c \ e^{(-\alpha a)}$$
(a. 16)

Boundary condition x = -aContinuity equation is $E_1(x_1) = E_2(x_1)$, then

$$A\cos(\gamma a) - B\sin(\gamma a) = d e^{(-\alpha a)}$$
(a. 17)
$$\frac{\partial E_1(x_1)}{\partial x_1} = \frac{\partial E_2(x_1)}{\partial x_1}$$

then

if

$$\gamma A \sin(\gamma a) + \gamma B \cos(\gamma a) = -\alpha \ d \ e^{(-\alpha a)}$$
(a. 18)

Substitute eq (a. 14) and (a. 17),

$$A\cos(\gamma a) + B\sin(\gamma a) = c e^{(-\alpha a)}$$
$$A\cos(\gamma a) - B\sin(\gamma a) = d e^{(-\alpha a)}$$

Adding above equation,

$$2A\cos(\gamma a) = (c+d)e^{(-\alpha a)}$$
(a. 19)

Substitute eq (a. 16) and (a. 18),

$$-\gamma A \sin(\gamma a) + \gamma B \cos(\gamma a) = -\alpha \ c \ e^{(-\alpha a)}$$
$$\gamma A \sin(\gamma a) + \gamma B \cos(\gamma a) = -\alpha \ d \ e^{(-\alpha a)}$$

Subtracting above equation,

$$2\gamma A \sin(\gamma a) = \alpha (c+d) e^{(-\alpha a)}$$
 (a. 20)

We can divide eq (a. 20) and (a.19)

$$\frac{2\gamma A \sin(\gamma a)}{2A \cos(\gamma a)} = \frac{\alpha (c+d) e^{(-\alpha a)}}{(c+d) e^{(-\alpha a)}}$$
(a. 21)

then,

$$\tan(\gamma a) = \frac{\alpha}{\gamma}$$
 (a. 22)

where $a = \frac{h}{2}$

In complete equation can be written,

$$\tan\left(\left(n_{1}^{2}k_{0}^{2}-n_{eff}^{2}k_{0}^{2}\right)\frac{h}{2}\right)-\left(\frac{n_{eff}^{2}k_{0}^{2}-n_{2}^{2}k_{0}^{2}}{n_{1}^{2}k_{0}^{2}-n_{eff}^{2}k_{0}^{2}}\right)=0$$
 (a. 23)

using the numerical method of the equation above, the effective value of refractive index could be determined.

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```
if (fa*fb > 0)
                          fprintf('Input a and b are out of interval\n');
                         else
while 1
                      if (abs (b-a) \le tol)
                      break
end
                         c = (a+b)/2;
                         fc = feval(f,h,c);
If (c = a | c = b)
                         fprintf ('maximum possible precission achieved\n');
                         break
end
0\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/_00\!\!/
                      if (fa*fc > 0)
                                             a = c;
                                             fa = fc;
                                             else
                                                  b = c;
                                                                   fb = fc;
                                             end
                      end
                      fprintf ('Neffective value = %18.9f\n', b);
end
%Execution program
function y=f(h/x)
y=tan((h*pi/1.55)*sqrt(1.468^{2}-x.^{2})) -
                       sqrt(x.^2-1.458^2))/(sqrt(1.468^2-x.^2));
```

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