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Cite as: AIP Conference Proceedings **1868**, 050021 (2017); https://doi.org/10.1063/1.4995148
Published Online: 04 August 2017

Yoppy Wahyu Purnomo, Chairunnisa Widowati, Tian Abdul Aziz, and Puri Pramudiani

Fractions Division Knowledge of Elementary School Student: The Case of Lala

Yoppy Wahyu Purnomo[1, a)], Chairunnisa Widowati [2, b)], Tian Abdul Aziz [1, c)] and Puri Pramudiani [1, d)]

[1]*University of Muhammadiyah Prof. DR. HAMKA, Jalan Tanah Merdeka, Rambutan, Pasar Rebo, Jakarta Timur*
[2] *Sekolah Dasar Islam SK Little Earth, Jalan Cipedak V, Srengseng Sawah, Jagakarsa, Jakarta Selatan*

[a)] yoppy.w.purnomo@uhamka.ac.id
[b)] Chairunnisawidowati@gmail.com
[c)]tian_aziz@uhamka.ac.id
[d)] puri.pramudiani@uhamka.ac.id

Abstract. Division of fractions is often acknowledged by mysterious rule which is not based on conceptual knowledge. The purpose of the study was to explore elementary school student's knowledge of division fractions. For this purpose, a case study was conducted. The participant of the study was Lala (pseudonym) who enrolled at one elementary school in East Jakarta. The data were collected by administering written test and semi-structured interview respectively. The findings of the study indicated that Lala was able to describe strategy of division fractions as inverse of repeated addition flexibly. She also had basic understanding of fractions division concept as equal sharing, but when she was challenged with advance problems, she performed poorly. Lala also encountered difficulty when dealing with dividing fraction by fraction problem in which she interpreted it as subtraction problem. In this case, her procedural knowledge was likely to be more salient than her conceptual knowledge.

INTRODUCTION

Fractions and its operation are cornerstone of mathematical knowledge for students at primary school as it is foundation for comprehending further mathematical concepts such as measurement, ratio, functions, and so forth. It is also inherent part of human life in which daily problems that are encountered cannot be represented solely with integer form. For instance, in case a certain number of things are going to be distributed to a number of people equally in which a number of people exceed that of the things, then concept of fractions is wielded.

Many rresearchers give special attention to the importance of developing competencies to deal with fractions [1–3] as it plays important role in everyday life. However, concept of fractions is likely to be difficult concept for students in primary school level. Besides, it is worsened with primary school teachers who have misconceptions and difficulties to teach fractions. In most cases, the topics of fraction tend to be taught meaninglessly by focusing merely on paper and pencil algorithm [1,4,5]. Moreover, teachers' lack of knowledge related to selecting and applying proper media or manipulatives for facilitating students' learning is perceived as another obstacle in teaching fraction.

National Curriculum Standards for Primary Education suggested that teaching fraction should be pointed towards developing competence in applying it in straightforward contexts and supported by models or manipulatives [6]. Meanwhile, in the current Indonesian curriculum, fractions is firstly introduced for students in the third school year of primary school in the way of presenting the simplest form of fraction such as ½, ¼, etc. Subsequently, fourth grade students are introduced to operation of addition and subtraction fractions with like denominators. Then, addition and subtraction of fractions with unlike denominators, and multiplication and division of fraction are taught

The 4th International Conference on Research, Implementation, and Education of Mathematics and Science (4th ICRIEMS)
AIP Conf. Proc. 1868, 050021-1–050021-8; doi: 10.1063/1.4995148
Published by AIP Publishing. 978-0-7354-1548-5/$30.00

for fifth graders. In general, the concept of fractions in primary schools tends to be presented by considering part-to-whole approach.

Fractions division become interesting content in elementary mathematics to be explored. This is because common strategies of division fractions which is used in most primary school mathematics workbooks is invert and multiply algorithm. It might be one of "mysterious" rule in basic mathematical operations and makes fractions division concept become more mechanistic and difficult to be understood by primary school students. That explanation can be referred to the transition from certain context into symbolic rules that occurs fast [1,7,8]. In addition, fractions division are taught by teaching strategies which emphasize heavily on memorization of procedures and rigid rules and put conceptual understanding aside. Nevertheless, several researchers argue differently that invert and multiply algorithm is more efficient and more closely linked to algebraic thinking [9]. Similarly, several researchers argue defferently about which type of knowledge (between conceptual and procedure) develops first or is more important [10,11].

Considering to the issues about various strategies in solving problems of fractions division, in this study we decide to investigate students' knowledge of division of fraction. In National Research Council [12], it was stated that understanding the nature of expertise can shed light on what successful learning might look like and help guide the development of curricula, pedagogy, and assessments that can move students toward more expert-like practices in a subject area. Therefore, the aforementioned explanations lead us to conduct a research which aims to analyze fifth grade students' conceptual knowledge of division of fraction. The research question is *"To what extent do students understand the concept of division of fractions?*"

METHODS

This research used case study to explore primary school students' conceptual knowledge of fraction division. A fifth grade student whose name is Lala (pseudonym) is selected purposively. She is a student in one of primary schools in East Jakarta. This study was conducted on the second semester in 2015/2016 academic years. Processes of data collection involve administering written test, interviewing, and analyzing student's worksheets. Lala is one of fourty students who participated in written test and of three students who participated in interview. Due to her interesting responses when she was interviewed, Lala was chosen.

Written test consisted of three fractions division questions. Each of them represented different cases, to wit: (1) division of natural number by fractions; (2) division of fractions by natural number, and; (3) division of fractions by fractions. The three questions are presented in findings parts. The questions proposed in interview part were developed according to students' responses on written test. Data obtained from the interview was transcribed and coded. Triangulation was performed through matching the data from students written test, transcipts, and worksheets. Student's worksheet was obtained during interview.

FINDINGS AND DISCUSSION

Generally, Lala completed the written test quite satisfactorily in which she was able to answer two of the three items correctly. Analysis of her writing responses, interviews, and worksheet was conducted to understand her conceptual knowledge about fractions division. Analysis of each item is analyzed in separate manner in the following explanation.

Division of Natural Number by Fractions Case

The first question in the written test is related to case of dividing natural number by fractions. This case was interpreted as counting the number of denominator subtracted from the natural number (dividend). It was in line with what several researchers called that division was comprehended as repeated subtraction or measurement concept [1,4,7,13]. Explanation with respect to the question and Lala's response is as follows

Question: At the birthday party, *two cakes will be shared equally among yourfriends. Each of them will get a half piece of a cake. Explain the way how much your friends could get from that cakes?*

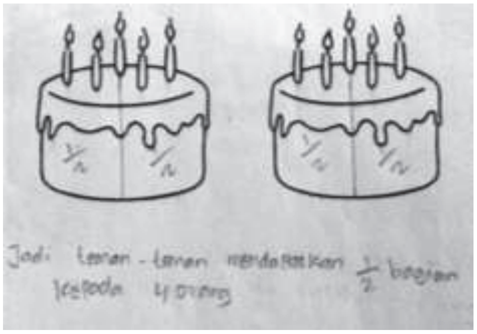

FIGURE 1. Lala's response on written test

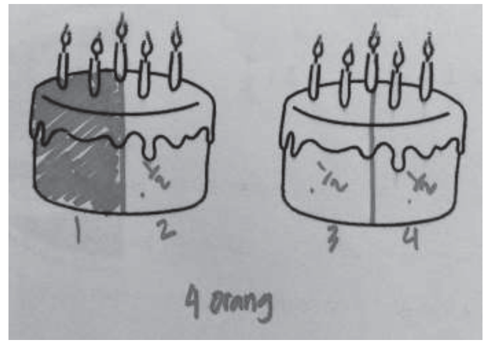

FIGURE 2. Lala's response when interviewed

According to Figure 1, Lala divided the two cakes into four similar pieces in which each piece was represented as ½. That answer could be interpreted as the number of times we can subtract ½ (a half of a cake) from two cakes before we get to 0 (the cakes were spent). However, this interpretation was not in line with Lala's response when she was interviewed. The following is Lala's excerpt that represents her way of thinking related to the first question.

Lala : *Each cake is divided into two half pieces. Then, when it was already sliced like this, it was cut into 4 pieces. This one (pointing to the shaded piece) is for 1 person. So, totally four.*
Researcher : *What did you mean by four?*
Lala : *four pieces (pointing to a half piece)*
Researcher : Then, *suppose that you have 12 doughnuts which will be shared to your friend equally and, each of them would get 2 doughnuts. How do you deal with this situation?*

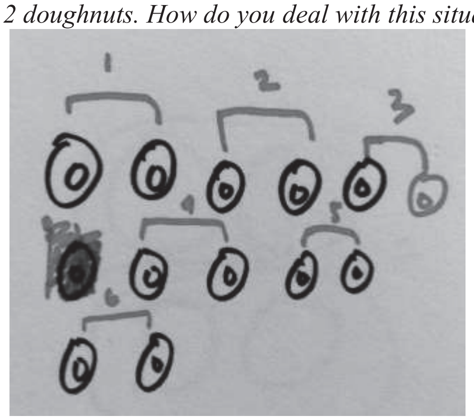

FIGURE 3. Lala's counting on strategy

Lala : *This is a pair for one person, two for this, this too, this (drawingcircles for two objects).*
Lala : *so there are six persons who got two doughnuts equally.*

With the same question in the interview session, Lala answered the division of two by halfcases in the way of counting how much half of piece that could be acquired from two cakes. Lala's division concepts was likely to be in parallel to idea of division as repeated subtraction. strong with an idea how much same parts which covered completely. This idea could be clearly seen when Lala was given a new problem related to the natural number division. Lala was asked to determine the number of friends that would get two doughnuts, if there were twelve doughnuts. Lala was counting on two for each person so it end up to 12 doughnuts and stop at the 6th person. (see Figure 3).

According to Lala's respond above, Lala was likely to hold division concept as inverse of addition. Systematically, Lala's idea related to division of natural number by fractions could be explained below.

$a \div \frac{1}{k} = n \leftrightarrow a = \underbrace{\frac{1}{k} + \frac{1}{k} + \cdots + \frac{1}{k}}_{n}$, with a as a divident and $\frac{1}{k}$ as a divisor.

When students encounter problems with fractions as a divisor, they tend to take concept of division as inverse of repeated addition into consideration as it might be easier to imagine than division concept as repeated subtractions. It might be that students seemed to have more intuitive knowledge to addition of fractions than subtraction of fractions. Determining the number of times in which a half could be added in order to get four and determining the number of times we can subtract ½ from two in order to get zero are different idea. It seems that the first one is easier than the latter.

Division of fractions by natural number case

Division of fractions division by natural number cases could be interpreted as the number of how many parts in each group which can be equally shared a known object. This concept is mostly known as fair sharing concept or division by grouping [1,4,7,13]. For this case, the question and Lala's response on written test can be seen below.

Question: Suppose that *you have a half of a pizza which will be shared to your friends equally and each of them will get half of itpart. Explain the way of getting the number of your friends that could get from that cake?*

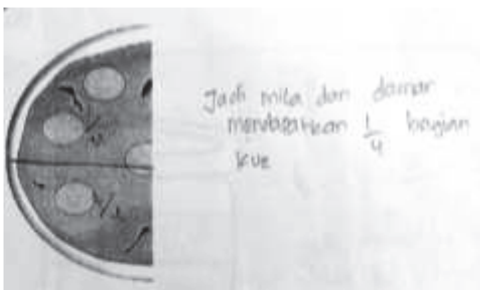

FIGURE 4. Lala's response on written test

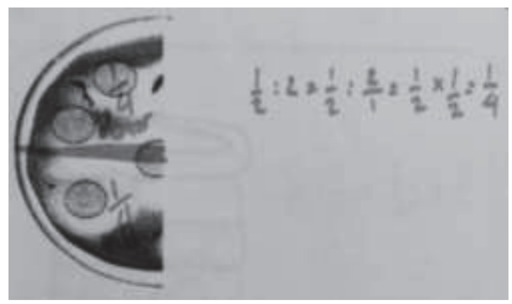

FIGURE 5. Lala's response when interviewed

According to Lala's worksheet on Figure 4. Lala answered with slicing a half of a pizza into two equal parts and wrote ¼ on each parts. Eventhough Lala's answer represented equal sharing concept, but when it was verified on the interview, Lala used her procedural knowledge to deal with the question. The following is Lala's expert in the interview.

Researcher	*: Lala, your father has a ½ pizza which will be shared to you and your sister equally. How many parts will you get?*
Lala	: ¼
Researcher	: *Why and how do you know?*
Lala	: *½ was divided into two slices so it equals to ¼ (pointing to procedure that Lala has done). Oops, it's divided by two, right (stop for a while, thinking again) …… a ½ right?*
Researcher	: What did you mean by ½?
Lala	: *This pizza was divided by 2. So, the result is ½, isn't it??*
Researcher	: *Why was ½ as the answer?*
Lala	: *Because mine is 1 part, and the two from these two slices*

Figure 5 showed that Lala used invert and multiply strategy to solve the second case. Lala's response was correct, so was its procedure. However, Lala expressed her doubt about the answer that she gave. Lala was likely to consider that each number which is divided by two is a half. This finding indicated that Lala's procedural knowledge is not compatible with proper conceptual knowledge. Several researchers were in common argument that both procedural knowledge and conceptual knowledge were important aspects to observe and evaluate mistakes when solving problems. Focusing only on procedural knowledge might impede development of intuitive sense and conceptual knowledge itself [1,5,14].

Several further questions were proposed to investigate division concept more deeply. These questions were related to equal sharing concept or grouping. The following is Lala's excerpt from the interview.

Researcher : *Lala, suppose that you have 12 doughnuts that you want to share totwo person equally, how do you share it?*

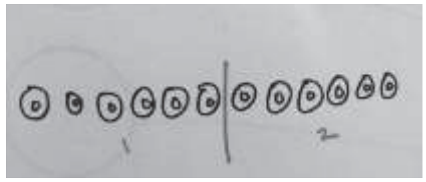

FIGURE 6. Lala's response related to partitioning concept

Lala : *For two persons? These are divided (grouping to) by two. So, six for each person*
Researcher : *Ok…. (proposinga new question related to equal sharing concept)*
Researcher : *Lala, could you explain what A, B, and C represent for?*

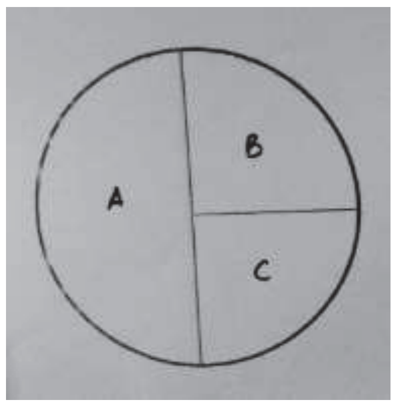

FIGURE 7. Lala's response related to partitioning concept

Lala : *Part B and C are same, they are 1/3. And part A is 2/3 because it is the biggest*

According to the excerpt above, Lala shared the 12 doughnuts by applying the equal sharing concept or partitioning or grouping, that is, grouping into two equalparts (see figure 6). On Figure 7. Lala was asked to mention fractions that represented A, B, and C. Lala answered 1/3 for the piece of B and C because there were three pieces totally and only one piece that was asked. Based on the same reasons, Lala answered 2/3 for A piece because that piece is combination of two pieces (B and C). Lala was likely to understand the basic of the equal sharing concept. However, she encountered difficulty, when she was given more challenging questions. Lala seemed to have misconception about the concept. It was also verified using further questions.

Researcher : *Imagine that I have 5 cakes, and I want to share it to four persons equally. Could you explain how you share it?*

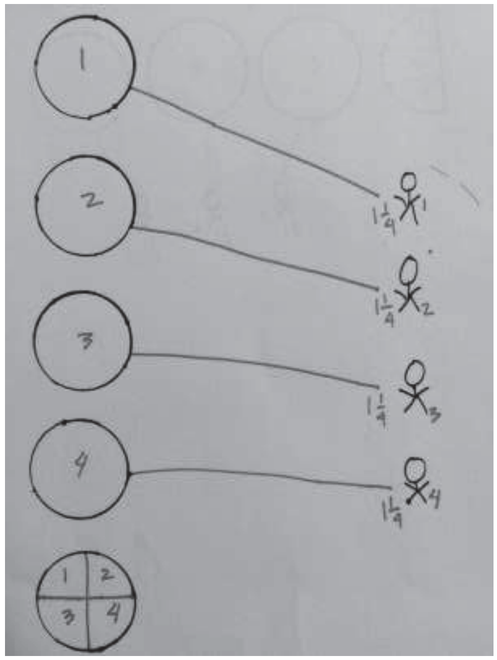

FIGURE 8. Lala's response is correct

Lala : *Each of them will get 1 whole cake. And the rest of cake is sliced into four pieces just like this. For example, I mark this 1, 2, 3, 4. So you just have to add 1 whole cake with this, so each person will get 1 ¼*

Researcher : *Ok… (proposing further question)*

Researcher : *Then if you have a 3 ½ cake, and you want to share it to person equally how do you share it?*

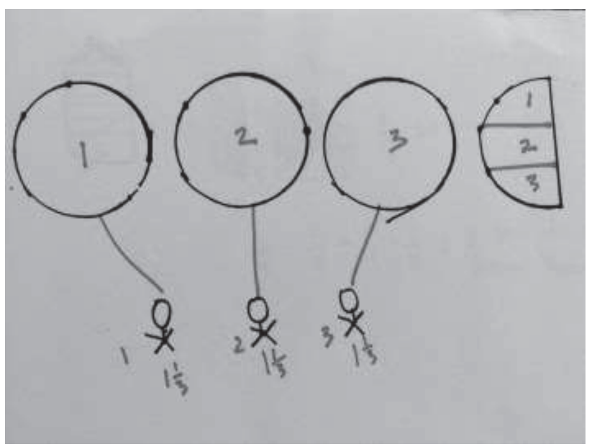

FIGURE 9. Lala's response is incorrect

Lala : like that, this one is sliced into three. Because each person gets 1. Then this one sliced into 3. That could be $1\frac{1}{3}$

The questions above showed that Lala had basic knowledge about equal shared concept. Lala was likely to be able to solve general problems about division. However, Lala encountered difficulty when she was given new problem. This finding indicated that Lala was likely to hold division concept as fair sharing limitedly. Fair sharing concept in fractions is a cornerstone for further learning about fractions.

Division of Fractions by Fractions Case

Just as the case of division of natural number by fractions, the case of division of fraction by fraction which involves fractions as divisor could be solveby means of repeated subtraction concept. Thus, the case of $\frac{3}{4} \div \frac{1}{4}$ can be solved by repeatedly subtracting $\frac{1}{4}$ from $\frac{3}{4}$ until it's all done. The number of repetition is the answer. The following are the question and Lala's answer in this case.

Question: *Ms. Vivi has ¾ kg flour which will be used to make doughnuts. In making a doughnut, Ms. Vivi spends ¼ kg. How do you know how many doughnuts that Ms. Vivi could make?*

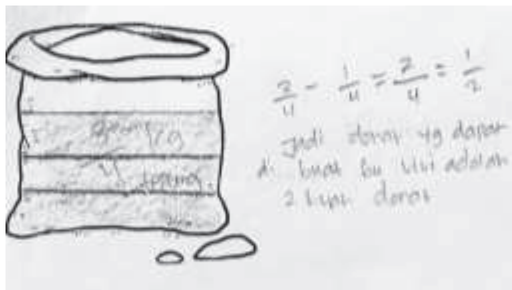

FIGURE 10. Lala's answer in written test

According to Lala's answer on figure 6, Lala applied the concept of division as repeated subtraction in which she conducted subtraction, that is $\frac{3}{4} - \frac{1}{4}$. This strategy could be explained differently. First of all, knowing that there were¾ kg and ¼ kg it which were used led her to conduct subtractions ¼ from ¾. Secondly, lala was likely to be confused with the last sentence in the question. The same answer occured when she was interviewed.

Researcher **:** *Ms. Vivi has ¾ kg flour which will be spent to make doughnuts. In making a doughnut, Ms. Vivi needs ¼ kg. How do you know how many doughnuts that Ms. Vivi could make?*

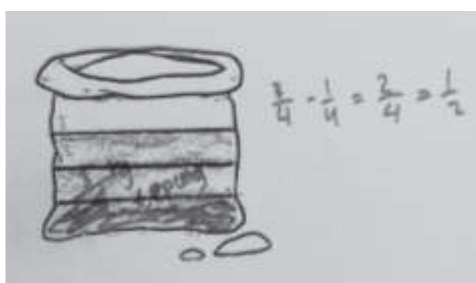

FIGURE 11. Lala's work on case $\frac{3}{4} \div \frac{1}{4}$ when she was interviewed

Lala : $\frac{2}{4}$ *doughnuts that Ms. Vivi could make*
Reseacher : *why did you answer like that?*
Lala : *because ¾ subtracted from ¼ equal* $\frac{2}{4}$
Researcher : what does $\frac{2}{4}$ mean?
Lala : $\frac{2}{4}$ *doughnuts*

Different from Lala's proper comprehension when dealing with division of natural number by fraction, Lala was likely to encounter difficulty when she dealt with division of fraction by fraction case (the case of $\frac{1}{a} \div \frac{1}{k}$, with a and

k is a natural number). Lala misinterpreted the presented questions and encountered difficulty to a division case which dividend and divisor are fractions.

It is important to develop ability to make sense of division of fractions problems in our daily life. The reason lays in the fact that students' ability in solving fraction division problems, particularly word problems, is in need of improvement. In most mathematics classrooms, fractions is taught by means of various instructional strategies that put conceptual knowledge aside. As a result, students encounter difficulty in dealing with their daily life problems. In addition, they tend to be exposed heavily with symbolic rule devoid of understanding. In this case, Philipp suggested that initially students should develop their understanding about the concepts of symbols and procedure before they apply it further [15].

CONCLUSION

This found indicated that students in primary school on this observed have basic knowledge about fractions division. Student's believe that division which involving a fraction as divider was the inverse from repeated addition, however when the division involving a fraction as divider or divided number, students experiencing a misinterpreted and misconception with assume it as subtraction case. They also had basic understanding of fractions division concept as equal sharing, but when they were challenged with advance problems, they performed poorly.

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